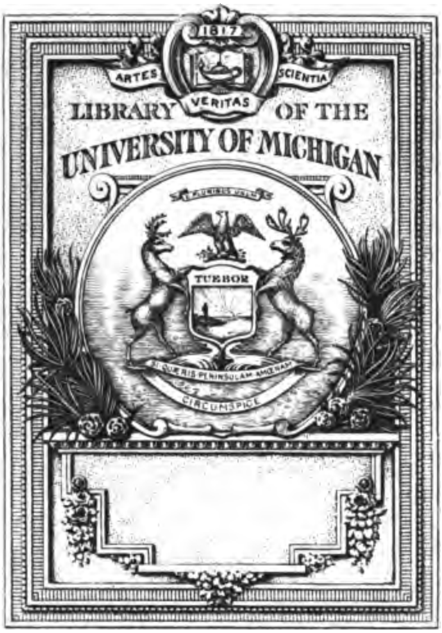
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CIRCUMSPICE

A

TREATISE

ON THE

NEW METHOD OF LAND SURVEYING,

WITH

THE IMPROVED PLAN OF

KEEPING THE FIELD BOOK;

DESIGNED

FOR THE USE OF SCHOOLS, AND FOR THOSE WHO WISH TO BE PRACTITIONERS OF THE SCIENCE.

BY THOMAS HORNBY,

LAND SURVEYOR.

LONDON:

PRINTED FOR BALDWIN, CRADOCK & JOY; AND HAMILTON, ADAMS & CO.; ALSO, FOR ISAAC WILSON, HULL; AND SOLD BY ALL BOOKSELLERS.

1827.

I. Wilson, Printer, Hull.

PREFACE.

It is the common practice of authors, in the first place, to dedicate their works to some man of rank or influence, and in the next, to present their readers with a long historical sketch of the rise and progress of the science which forms the particular topic of investigation. As such a custom can add little value to a book, which is not otherwise possessed of merit, I shall adopt neither of these methods on the present occasion; but merely confine myself to stating, by way of preface, what are the contents of the following sheets.

The Work consists of two Parts,

Part I. contains the Mensuration of right Lines and plain Figures, and is divided into two Sections.

Section I. contains Definitions and practical Problems in Geometry, (selected from Euclid, Simpson, Emerson, Hutton, and others,) necessary to be known by every Surveyor.

INDEX.

PART I.

ERRATA.

PAGE

5 Last line, for AC read BC.

16 Line 4, for many parts, read many equal parts.

38 Line 8 from bottom, for $\frac{AB+EC}{2}$ read $\frac{AB \times EC.}{2}$

40 Line 2 from bottom, $\frac{\sqrt{AB^2+CD^2}}{2} = \frac{\sqrt{30^2+20^2}}{2}$ read $\sqrt{\frac{AB^2+CD^2}{2}}$ $= \sqrt{\frac{30^2+20^2}{2}}$.

52 Line 6, for *ik*, read *no*.

74 After Example, read Reduce 3R. 25P. to the decimal of an acre.

83 Line 7, for preceding read following.

94 Line 7, for part 2 read part 1.

145 Line 17, for S——t read York.

146 Line 10, for ⊙ 2 read ⊙ 22. Lines 11 and 12, from ⊙ 11 to ⊙ 23 read from ⊙ 23 to ⊙ 11. Line 28, for △ b read △ c.

147 Line 23, for to ⊙ 1 read to the △ S of ⊙ 1.

174 Bottom line, for Prob. 12 read Prob. 9th.

176 Line 4 from bottom, for $\frac{4}{3}\sqrt{dx)} \times \frac{n+m}{2}$., read $\frac{4}{3}\sqrt{dx)} \times \frac{4x}{10} \times \frac{n+m}{2}$.

186 Bottom line, for *t g h t* read *t g h D*.

195 Line 1, for Prob. 6, Part 2, read Prob. 6, Section 2, Part 1.

200 Line 6 from bottom, read from one ⊙, &c.

238 Line 15, after Section 2, Part 2, read and Prob. 4, Section 2, Part 1.

242 Line 10, for field read fixed.

Note.—In the Field Book to Plate 4, page 3, the line from ⊙ 23 to ⊙ 11, should follow the line from ⊙ 22, to ⊙ 10, page 2, according to the directions laid down page 146, which wrong entry, (or that of any other line,) does not, in the least, affect the truth of the work, provided it is truly measured and entered at any time during the survey. But every beginner should carefully observe the directions until he is entirely master of the business.

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LAND SURVEYING.

PART I.

Of Geometrical Definitions and Problems, with the Mensuration of right lined and circular Figures.

SECTION I.

OF GEOMETRICAL DEFINITIONS AND PROBLEMS.

FIRST.—DEFINITIONS.

1. A point is that which hath no parts or magnitude.

2. A line is length without breadth or thickness; as the line A B.

A ———————————— B

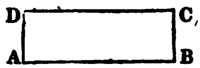
3. The ends or extremities of a line are points.

4. A straight line is that which lies always in the same direction, between its extreme points; as the line B C.

B ———————————— C

5. A curve line is a line that continually changes its direction between its extremities; as the curve A B.

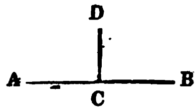
6. A superficies is a figure of two dimensions, viz. length and breadth; as the parallelogram A B C D.



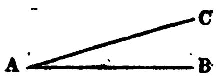
Hence the extremities of a superficies are right lines.

7. Parallel lines are lines, whether straight or regularly curved, that never meet or intersect each other, though ever so far produced; as the lines A B and D E.

8. A perpendicular is a straight line standing on another straight line, in such a position as to make its adjacent angles equal to each other; as C D on A B.

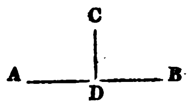


9. An angle is the inclination of two lines, having different directions, which meet together in a point, called the angular point; as the two lines A B and A C.

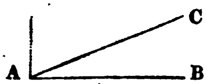


10. A right angle is that which is made by one right line standing perpendicular upon another, or when the

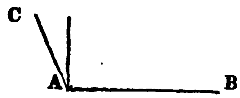
angles on each side of a line, that stands upon another, are equal; as the angle A D C=B D C, made by the line C D, standing upon A B.



11. An acute angle is less than a right angle; as the angle B A C.



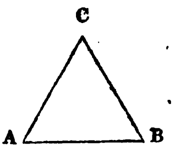
12. An obtuse angle is greater than a right angle: as the angle B A C.



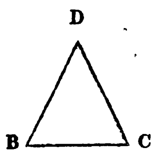
Note.—When three letters are set to denote an angle, it is to be understood that the middle letter stands at the angular point.

13. A figure of three sides is called a triangle, and is named according to the relations of its sides and angles.

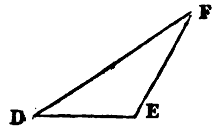
14. An equilateral triangle, is one whose sides are all equal; as the triangle A B C.



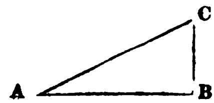
15. An isosceles triangle is that which has only two of its sides equal; as the triangle B C D.



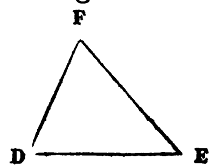
16. A scalene triangle is that which has all its sides unequal; as the triangle D E F.



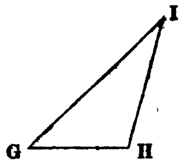
17. A right angled triangle is that which has one right angle; as the triangle A B C.



18. An acute angled triangle, is that which has all its angles acute; as the triangle D E F.

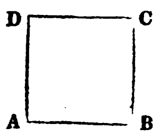


19. An obtuse angled triangle is that which has one angle obtuse; as the triangle G H I.

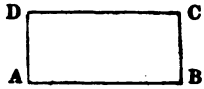


20. A four-sided figure is called a quadrangle, or a quadrilateral; which has either all its sides equal and parallel, or both pairs of opposite sides equal and parallel; such as a square, parallelogram, rhombus, or a rhomboid.

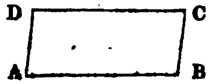
21. A square is an equilateral quadrangle, having all its sides equal, as well as its angles, which are right ones; as the square A B C D.



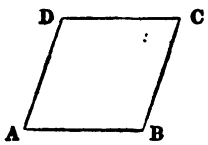
22. A parallelogram is a quadrilateral which has all its angles right ones, and each of its pairs of opposite sides parallel; as the parallelogram A B C D.



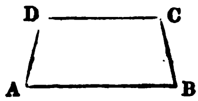
23. A rhomboid is a parallelogram, having its opposite sides equal to one another, but its angles oblique ones; as the rhomboid A B C D.



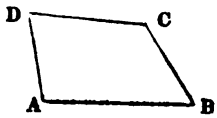
24. A rhombus is an equilateral rhomboid, having all its sides equal, but its angles oblique ones; as the rhombus A B C D.



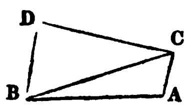
25. A trapezoid is a four-sided figure, having one pair of its opposite sides parallel; as the trapezoid A B C D.



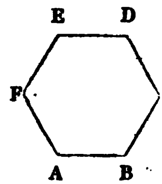
26. All other four-sided figures are called trapeziums; as the trapezium A B C D.



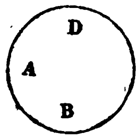
27. A diagonal is a straight line joining any two opposite angles of a four sided field; as the diagonal A C.



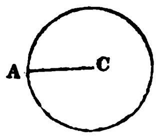
28. A regular figure is that which has all its sides and all its angles equal; and is generally called a regular polygon, when more than four sides; as the hexagon A B C D E F.



29. A circle is a plain figure, bounded by a curve line, called the circumference; which circumference is every where equi-distant from a point within it called the centre; as the circle A B D.

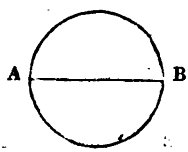


30. The radius of a circle is a straight line drawn from the centre to the circumference; as the line A C.

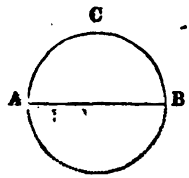


Hence, the radius is the line or distance that describes the circumference.

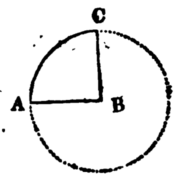
31. The diameter of a circle is a straight line, drawn through the centre, cutting the figure into two equal parts; as the line A B.



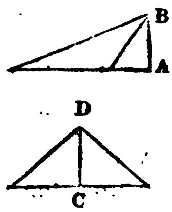
32. A semicircle is half of a whole circle; as the semicircle A B C.



33. A quadrant, or quarter of a circle, is half of a semicircle; as the quadrant A B C.



34. The altitude of any figure is a perpendicular from its vertex, to its opposite side or base, or to the opposite side produced; as the perpendicular A B or C D.



GEOMETRICAL PROBLEMS.

PROBLEM 1.

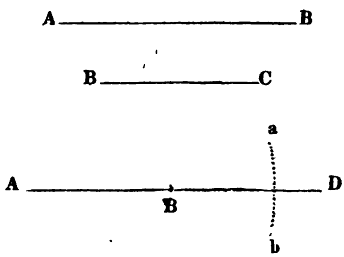
To draw a straight Line between two given points, A and B, upon a plane.

Apply the edge of your scale or ruler to the two points; by which draw the line A B. and it will be the line required.

PROBLEM 2.

To produce a given Line, A B, that is too short; to a given distance, B C.

Apply the edge of your scale along the line A B; by which draw or produce from the point B, a line of any length, greater than the line to be produced. Then, with the radius B C and centre B, describe the arc a b, to intersect the line produced; and it is done.

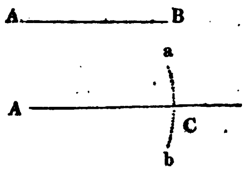


PROBLEM 3.

From a given point A in a given right Line, to lay off a line or distance A B.

With the radius A B and centre A, describe the arc a b; then will the distance between the point A, and point of intersection C, be equal to the given line A B.

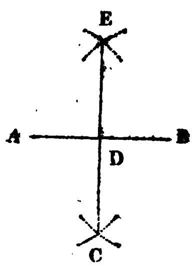
Or apply your scale to the line A. and make A C=A B.



PROBLEM 4.

To divide a given Line A B, into two equal parts.

From the ends A B, as centres, with one and the same extent, (but greater than half the line A B,) describe two arcs to intersect each other in C, and E,: draw E C to cut the line A B, in D; then will the line A B, be divided into two equal parts, (i. e.) A D=B D.

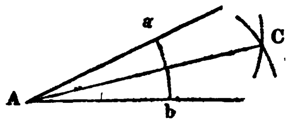


PROBLEM 5.

To bisect an Angle A, (i. e.) to divide it into two equal parts.

With any radius and centre A, describe the arc a b; then from the centres a and b, with one and the same ra-

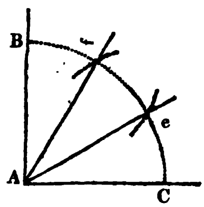
dius, describe two arcs intersecting each other in C; draw A C, then will the angle be divided.



PROBLEM 6.

To divide a given right Angle into three equal parts.

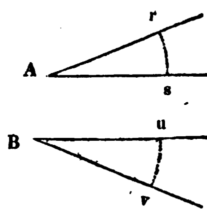
With any radius and centre A. decribe the arc B C, then with the centre C, and the same radius, describe the arc f. Also with the centre B, and the same radius, describe the arc e; draw A f, A e, and it is done.



PROBLEM 7.

To make an Angle B equal to a given Angle A.

With the centre A, and any radius, describe the arc r s; then draw any line, B C, and from B, with the same radius as before, describe the arc u v, make u v=r s, and draw B,v. Then will the angle B, be equal to the angle A.



Cor.—When an arch, u v, is described with a radius of 60° taken from a scale of chords, an angle of any magnitude is constructed thus. Take the number of degrees the

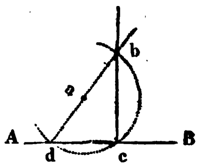
angle is to contain from the same scale, and apply them from u, to v. Draw B v, and it is done.

Note.—An angle of more than 90° is taken off at twice.

PROBLEM 8.

To raise a Perpendicular from a given point C, in a given line A B.

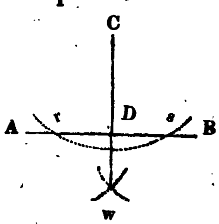
With any centre, as a, and radius a c, describe an arc cutting the given line AB, in c, and d. Through the points d, and a, draw a line, cutting the arc in b. Lastly, draw b, c, and it will be the perpendicular required.



PROBLEM 9.

To let fall a Perpendicular, from a given point C, upon a given right line A B.

With the centre C, and any radius greater than C D, describe an arc, to cut the line A B, in r and s. Then, with the centres r and s, with any radius greater than half the distance r s, describe two arcs intersecting each other in w. Draw C, w, to intersect the line A B, in D; then will C D be the perpendicular required.

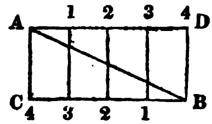


Note.—Short perpendiculars are more readily erected or let fall from a given point, by a scale or rule having a line cut across it perpendicular to its edge, by lying the line so cut in the direction of the line whereon the perpendicular is to be erected or let fall; and then drawing by the edge of the scale, from the given point, a straight line.

PROBLEM 10.

To divide a given Line A B, into any number of equal parts.

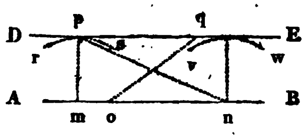
From the point A draw the line A D, at random. Then from the point B draw B C parallel to it. Next on each of the lines thus drawn, (viz. on A D and B C) set off the same number of equal parts the line A B is to be divided into, beginning at the points A and B. Join every two opposite points of division by right lines, then will the line A B, be divided into the number of equal parts required.



PROBLEM 11.

To draw a Line parallel to a given Line AB.

From any two points, as m and n, in the line A B, and with any radius, describe two arcs, as r s, v w.—Draw DE to touch without cutting the said arcs; and it will be the parallel line required.



Note.—When the parallel line is to be at a given distance from the given line, make the radius you describe the arcs with, equal that distance.

The same must likewise be done when the parallel line is to pass through a given point.

Note 2.—This Problem is always performed in surveying by the parallel ruler, and is both quickly and accurately done by drawing a line along the edge of it, after the said edge has been laid along the given line, as along the line AB, and then moved forwards the given distance, as the distance m p.

Otherwise, the above Problem may be done thus:—

From any point o, in the line A B, draw the line o q, and divide it into two equal parts; then from any other point n, in the line A B, draw n p through the point of bisection, or

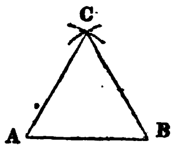
middle of o q, and make the distance of p equal that of n, from the said point; lastly, through the points q and p draw the line DE, and it will be the parallel line required.

The above is Ozanam's method, as laid down in his Recreations (by Dr. Hutton, vol. 1, p. 265) and is the best method I know of in setting out a line parallel to a given line, when wanted in any large survey (provided the points p and q lay near the extremities of the required line) by reason of the lines oq, np, crossing part of the interior fences, and making the lines wanted afterwards considerably shorter.

PROBLEM 12.

To make an Equilateral Triangle on a given right line, A B.

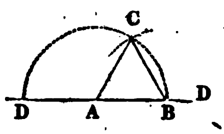
With radius the length of the line AB, and centres A and B, describe two arcs intersecting each other in C;—draw A C and BC, then will ABC be the triangle required.



PROBLEM 13.

To make a Triangle with three given right lines, A B, B C, and C A.

On any right line drawn at random, as D D, mark off the length of any one of the three given lines (as A B in this case); then from the centre A, with the radius A C, describe an arc. Also, from the centre B, with the radius B C, describe another arc, intersecting the former in C. Draw A C, and B C, then will A B C, be the triangle required.



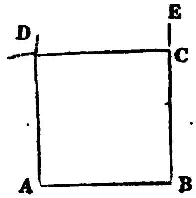
Note.—In the very same manner is an angle laid down, or constructed, that is taken with the chain, when the sides that contain it are unequal; (i. e.) when A B is greater, or less than A C. But when they are equal, with

radius A B, or A C, describe the semi-circle D C B; from B apply the chord B C, upon the circumference; afterwards draw A C, and it is done.

PROBLEM 14.

Upon a given right line, A B, to make a Square.

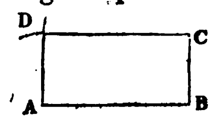
From the point B, erect the perpendicular line B E, on which make B C = A B. Then, with the radius A B, and centres C and A, describe two arcs intersecting each other in D. Draw A D and C D, and it is done.



PROBLEM 15.

To make a Rectangle, whose length, A B, and breadth, B C, are given.

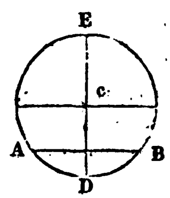
Draw C B perpendicular to A B; then, with the centre C, and radius A B, describe an arc. Also, with centre A, and radius B C, describe another arc to intersect the former in D. Draw A D, and D C: then will A B C D be the parallelogram or rectangle required.



PROBLEM 16.

To find the Centre of a given Circle.

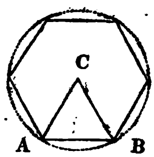
Draw any chord, as A B, and bisect it with the line E D, (by Problem 4); which bisecting line will be the diameter. Hence, bisect D E in the point c; then will c be the centre required.



PROBLEM 17.

To make any regular Polygon, on a given right line A B.

Draw A C and B C, so as to make the angle A and B each of them half the angle of the polygon. Then, with the centre C, (the intersecting point of the two lines A C and B C) and radius C A=C B describe a circle. Apply the side or line A B continually round the circumference, and it is done.



Note.—The angle of the polygon, (or double the angle C A B) is equal to the number of degrees contained in the angle at the centre, subtracted from 180°; (i. e.) it is equal 180°, minus 360° divided by the number of sides: which gives us the following table of degrees, contained in the angle at the centre, and the angle of the polygon, from 3 to 12 sides.

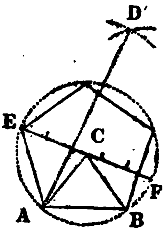
Name of the Polygon.	*No. of Sides.*	*Angle at the Centre.*	*Angle of the Polygon.*
Trigon	3	120°	60°
Tetragon	4	90	90
Pentagon	5	72	108
Hexagon	6	60	120
Heptagon	7	$51\frac{3}{7}$	$128\frac{4}{7}$
Octagon	8	45	135
Nonagon	9	40	140
Decagon	10	36	144
Undecagon	11	$32\frac{8}{11}$	$147\frac{3}{11}$
Dodecagon	12	30	150

PROBLEM 18.

In a given circle to inscribe a regular Polygon.

Make the angle C, at the centre, equal to the angle at the centre of the polygon contained in the third column of the above table. Then will the distance A B, be one side of the polygon; which side, being applied round the circumference its proper number of times, will inscribe the figure required.

COR.—Hence, the arc A B is one of the equal parts, which the circumference is divided into by so doing.



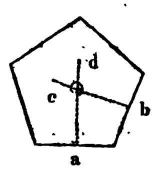
The same Problem may be done thus.

Draw the diameter E F, and divide it into as many parts as the figure is to have sides. Then, with the radius E F, from the centres E and F, describe two arcs crossing each other at D. From D draw a line through the second division on the diameter, until it cross the circumference at A. Then will A E be one of the sides of the polygon required.

PROBLEM 19.

To find the centre of any given regular Polygon.

Bisect any two of the sides; then, from the bisecting points a and b, draw the two perpendiculars a d, and b c, to intersect each other as in o. Then will o be the centre of the polygon.

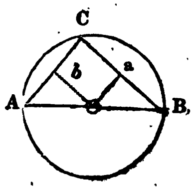


PROBLEM 20.

To describe a Circle about a given Triangle, A B C.

Bisect any two of the sides (as in the last problem); then, from the point of intersection, made by the bisecting lines crossing each other, as a centre, with the radius or

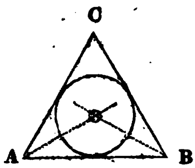
distance between the said point and any one of the angles, describe the circle, and it is done.



PROBLEM 21.

To inscribe a Circle, within a given Triangle, A B C.

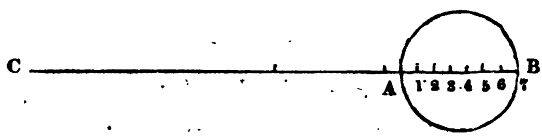
Bisect any two of the angles, as A and B, with the lines A O, and B O; and the point O, where the said lines cross each other, will be the centre of the circle. Then, with the centre O, and radius the nearest distance between the centre and any side, describe the circle, and it is done.



PROBLEM 22.

To draw a Right Line, equal to the circumference of a given Circle.

Produce the diameter A B, to any distance at random. Then make A C equal three times the diameter, and one-seventh part more, and it will be equal to the circumference required.

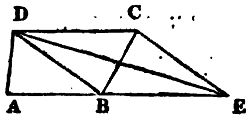


Hence a circle is equal to a right angled triangle, whose base is 3⅐ times the length of the diameter, and perpendicular equal to radius.

PROBLEM 23.

To make a Triangle, equal to a given Trapezium, A B C D, or take away an Angle from it, or any other four-sided Field.

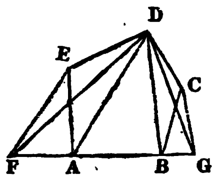
Produce the side A B, then draw the diagonal D B; also C E parallel to it, meeting the line produced in E. Draw D E, then will A E D be the triangle required.



PROBLEM 24.

To make a Triangle equal to the figure A B C D E A.

Produce the side A B both ways; then draw the diagonals A D and B D; also E F and C G parallel to them, meeting the lines produced in the points F and G. Draw F D, G D, then will F D G be the triangle required.



Note.—And thus, by taking away one angle after another, may any other straight-sided figure be reduced to a trapezium or triangle.

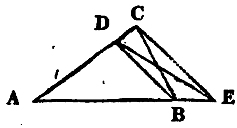
Note 2.—The two preceding Problems being founded on Euclid 37, and 38, 1st, it follows from thence, likewise, that by increasing or diminishing the base of any triangle, another of equal area may be drawn. Hence we have the following:

PROBLEM 25.

To reduce a given Triangle, A B C, to another of equal area, but of a different base.

Let A E be the proposed base, being in the same line A B; draw the line C E from the top of the given triangle to the point E, proposed; and through the angle B, of the given

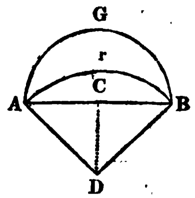
triangle, draw B D parallel to C E; next draw the line D E, then will the triangle A D E be equal to the triangle A C B, which is Emerson's method, Prob. 22, Book 8 Geo.



PROBLEM 26.

Upon a given Line, as the line A B, to describe the Lune of Hippocrates.

Bisect the given line A B in C; from C, demit the perpendicular C D, and make it equal to A C. Draw A D, D B; then, with the radius A C=B C, and centre C, describe the semicircular arc, A G B, and with radius A D= B D, and centre D, describe the quadrantal arc A r B; then will the space A G B r A, included between the two arcs, be the lune required.

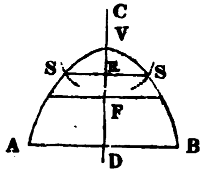


PROBLEM 27.

To construct a Parabola, having the absciss and ordinate, D V and A D, given.

Divide the square of the ordinate by four times the absciss, and the quotient will be V F, the focal distance—*Hutton's Mensuration, page* 355. Then, in the axis produced without the vertex V, take V C=V F, and draw several double ordinates, as S R S; then, with the radii C R and centre F,

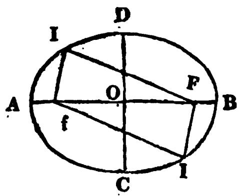
describe arcs cutting the corresponding ordinates in the points s, s. Draw a curve through all the points of intersection, and it will be the parabola required.



PROBLEM 28.

Given the two Axes A B and D C of an Ellipse, to describe the circumference A C I B D I A.

Draw the two axes at right angles, one to another, and make the point of intersection the centre of both; then take A O, equal to half the transverse axis, as a radius, and, with centre D, describe arcs cutting A B in the points f F, which points are the foci; next take a chord, the length of A B, and fix its two ends fast in the foci; then stretch the chord, and it will reach to I in the curve. Hence, by moving any pointed instrument round within the chord, keeping it always stretched, it will trace out the circumference required.



SECTION II.

OF THE AREAS OF RIGHT-LINED AND CURVED FIGURES.

The area of any straight-lined or curved figure is the measure of its surface, or the space inclosed by its boundaries, and is calculated by the number of squares contained in the space so inclosed: the side of each square, in land measure, being generally taken one link, or 7,92 inches in length: of course the area is said at first to be so many square links; but since the lengths of lines measured with the chain are always set down in links as integers, after the area is found in square links, by cutting off five figures to the right hand as decimals, and multiplying them by 4, and the product again by 40, cutting off in both cases five figures to the right hand; those on the left cut off at the three operations, constitute the area, and are then called acres, roods, and perches.

TABLE OF SQUARE MEASURE.

Sq. Links.	*Sq. Perches.*				
625	1	*Sq. Chain.*			
10,000	16	1	*Sq. Rood.*		
25,000	40	2.5	1	*Sq. Acre.*	
100,000	160	10	4	1	*Sq. Mile.*
64,000,000	102,400	6,400	2,560	640	1

PROBLEM 1.

To find the Area of a Parallelogram.

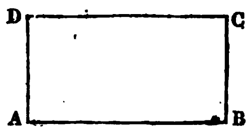
RULE.

Multiply the length by the perpendicular breadth, and the product will be the area.

Note.—The above rule is true, whether the figure be a square, rhombus, or a rhomboid.

EXAMPLE.

Required the area of the field, or paralellogram A B C D whose length A B is 640, and breadth A D 310 links.




```
      640
      310
     ----
     6400
    1920
   ------
   1.98400
         4
  --------
   3.93600
        40
  --------
  37.44000
  --------
```

Ans. 1A. 3R. 37P.

PROBLEM 2.

To find the Area of a Triangle.

RULE.

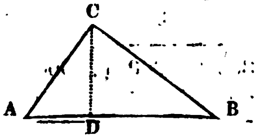
Multiply the base by the perpendicular, and half the product will be the area.

RULE II.

Add the three sides together, and take half the sum; subtract each side from that half sum; then multiply the said half sum and the three remainders continually together; and the square root of the product will be the area of the triangle.

EXAMPLES.

1.—Required the area of the triangular field A B C, whose base A B is 650, and perpendicular height D C, 430 links respectively.



650
430

19500
2600

½)2.79500 = double area.

1.39750
4

1.59000
40

23.60000

Ans. 1A. 1R. 23P.

2.—Suppose the sides A C, B C, and B A, are 300, 400, and 500 links respectively; required the area of the triangle.

300
400
500

½)1200

600 = half the sum of the sides.

```
      600          600          600
AC = 300     BC = 400     BA = 500
     ----         ----         ----
      300          200          100  the three differences.
     ----         ----         ----

           600
           100
        ------
         60000
           200
        ------
      12000000
           300
    ----------
    3600000000 (.60000
    36 . . . .       4
    --         -------
    ..00000000 2.40000
    ----------      40
               -------
               16.00000     Ans. 0A. 2R. 16P.
               -------
```

PROBLEM 3.

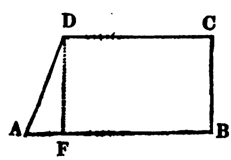
To find the Area of a Trapezoid.

RULE.

Multiply the sum of the parallel sides by the perpendicular distance between them, and half the product will be the area.

EXAMPLE.

Required the area of the trapezoid A B C D, whose parallel sides A B and D C are 800 and 640 links; also the perpendicular distance D F, 400 links.



	800
	640
Sum of the perpendicular sides =	1440
	400
	2)576000 Double area.
	2.88000
	4
	3,52000
	40
	20.80000

Ans. 2A. 3R. 20P.

PROBLEM 4.

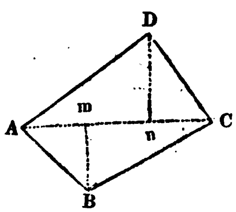
To find the Area of a Trapezium.

RULE.

Multiply the diagonal by the sum of the two perpendiculars, let fall upon it, from its opposite angles; and half the product will be the area.

EXAMPLE.

Required the area of the trapezium A B C D, when the the diagonal A C is 930 links, and the two perpendiculars B m and D n, 320 and 300 links respectively.



```
         320
         300
        ————
         620 Sum of the perpendiculars.
         930 Diagonal.
        ————
       18600
       5580
      ——————
   2)576600 = Double area.
      ——————
     2.88300
           4
      ——————
     3.53200
          40
      ——————
    21.28000
      ——————         Ans. 2A. 3R. 21P.
```

Note.—When the sides are in arithmetical progression, the area is equal to the square root of the continual product of all the four sides.—*Hutton's Mensuration, Cor.* 5, *page* 105.

PROBLEM 5.

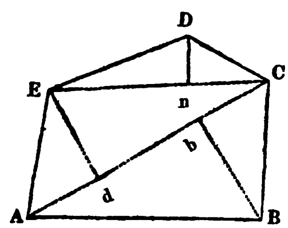
To find the Area of an irregular Field or Figure.

RULE.

Divide the figure into trapeziums and triangles by diagonal lines, and find the area of each piece separately. Then, the sum of all the separate areas will be the area of the whole irregular figure.

EXAMPLE.

Required the area of the irregular figure A B C D E A, whose diagonals and perpendiculars are as below.



$$\left.\begin{array}{l} AC = 1260 \\ Ed = 440 \\ Bb = 380 \\ Ec = 950 \\ Dn = 410 \end{array}\right\}\text{Links.}$$

$$\begin{array}{r} Ed = 440 \\ Bb = 380 \\ \hline 820 \\ \hline \end{array} \qquad \begin{array}{rl} AC = 1260 & \\ 820 & \\ \hline 25200 & \\ 10080 & \\ \hline 2)1033200 & \text{The double area.} \\ \hline 5.16600 & \text{Area of the trapezium.} \\ \hline \end{array}$$

$$\begin{array}{rl} EC = 950 & \\ 410 & \\ \hline 9500 & \\ 3800 & \\ \hline 2)389500 & \\ \hline 1.94750 & \text{Area of the triangle } E\,C\,D. \\ 5.16600 & \text{Area of the trapezium } A\,B\,C\,E \\ \hline 7.11350 & \\ 4 & \\ \hline .45400 & \\ 40 & \\ \hline 18.16000 & \\ \hline \end{array}$$

Ans. 7A. 0R. 18P.

PROBLEM 6.

To find the Area of a regular Polygon.

RULE.

Multiply the sum of its sides, by the radius of its inscribed circle, (or perpendicular demitted from the centre upon one of them,) and half the product will be the area.

RULE II.

Multiply the square of a side, by the multiplier standing opposite its name in the following table, and the product will be the area.

Names of Polygons.	*No. of Sides.*	*Area or Multipliers*	*Tangent of the ∠ CAD. or ½ the ∠ of the Polygon.*	*Radii of Circumscribing Circles.*	*Radii of Inscribed Circles.*
Trigon....	3	0.4330127	.5773502	.5773	.28867510
Tetragon..	4	1.	1.0000000	.7071	.5
Pentagon..	5	1.7204774	1.3763819	.8506	.68819095
Hexagon..	6	2.5980762	1.7320508	1.0000	.86602540
Heptagon...	7	3.6339124	2.0765213	1.1523	1.03826065
Octagon...	8	4.8284271	2.4142135	1.3065	1.20712075
Nonagon..	9	6.1818242	2.7474774	1.4619	1.37373870
Decagon...	10	7.6942088	3.0776835	1.6180	1.53884175
Undecagon.	11	9.3656399	3.4056872	1.7744	1.70284360
Dodecagon	12	11.1961524	3.7320508	1.9318	1.86602540

EXPLANATION OF THE TABLE.

1st.—The tangents of the angle C A D, or half the angle of the polygon, are taken from a table of natural tangents.

2nd.—The multipliers are found by multiplying the tangent of the angle C A D, by the number of sides, and taking one-fourth of the product, which multipliers are the areas of polygons, whose sides are unity or one.

3rd.—The radius of the circumscribing circle is found by dividing half the side (or .5) by the nat. sine of half the angle at the centre.

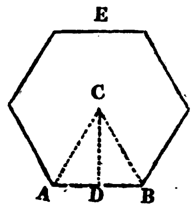
4th.—The radius of the inscribed circle is found by multiplying the tangent of C A D by .5, or half the side.

Note.—When the side of any other similar polygon is found, the radius of its circumscribing circle is had, by multiplying the said side by the tabular radius. For, by similar triangles, as the side unity is to the tabular radius, so is any other side to the radius of its circumscribing circle.

EXAMPLE.

Required the area of the regular hexagon A B E A, whose side A B is 430 links.

First 1.7320508 (tang. ∠ CAD) × 215 (AD) = 372.4 = CD the perpendicular.



Then by Rule 1.

```
    430
      6
   ----
   2580 = the sum of the sides.
   372.4
   -----
   10320
   5160
 18060
 7740
 -------
2)9607920
 --------
 4.803960
        4
 --------
 3.215840
       40
 --------
 8.633600
 --------  Ans. 4A. 3R. 8P.
```

By Rule. 2. 430 = side A B.

```
            430
         ------
          12900
         1720
         ------
         184900
      2.5980762  Tabular area.
      ---------
         369800
       1109400
      1294300
     14792000
    1664100
    924500
   369800
   ----------
   4.803842
          4
   --------
   3.215368
         40
   --------
   8.614720
   --------
```

Ans. 4A. 3R. 8P.

PROBLEM 7.

To find the Diameter and Circumference of a Circle, one from the other.

RULE I.

As 1 is to 3.1416, so is the diameter to the circumference; as 3.1416 is to 1, so is the circumference to the diameter.

RULE II.

As 113 is to 355, so is the diameter to the circumference; as 355 is to 113, so is the circumference to the diameter.

EXAMPLE I.

Required the circumference of a circle whose diameter is 7.

By Rule 1, $3.1416 \times 7 = 21.9912$, the circumference.

EXAMPLE 2.

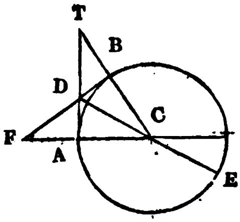
If the circumference of a circle be 355, what is the diameter?

By Rule 1, $355 \div 3.1416 = 112.9997$, the diameter, nearly.

Having shewn above, the proportions which the diameter of a circle bears to the circumference, according to the writers on Mensuration, I shall, in the next place, endeavour to shew how the number 3.14159, &c. or the circumference of a circle whose diameter is 1, is found, or approximated, from the radius or half the diameter; and in order to make every thing as clear as I can, I shall premise the following

LEMMA.

If AC be the radius of the circle ABEA, AB an arc, AT the tangent, and TC the secant of the same arc; and BF a tangent to the circle at B, then will $AD = \dfrac{AC \times TB}{AT}$.



For BD = DA (Euclid 37.3) consequently ECD bisects the arc AB. Hence by similar triangles AT : AC : : TB : (BD) $= DA = \dfrac{AC \times TB.}{AT}$ Q.E.D.

Hence if $r = AC =$ radius, $t = AT$ the tangent, then will $\sqrt{r^2 + t^2} = TC$ the secant, and $\sqrt{r^2 + t^2} - r = TB$, also $\dfrac{r\sqrt{r^2 + t^2} - r^2}{t} = AD$, consequently by taking $r = 1$, and the angle $ACB = 45^0$, the operation will stand as below.

$$\frac{\sqrt{1^2+1^2}-1}{1} = \frac{\sqrt{2}-1}{1} = .4142135623 = \text{tang. of } \tfrac{1}{2} \text{ the arc AB.}$$

$$\frac{\sqrt{1^2 + .4142135623^2}-1}{.4142135623} = .1989123685 = \text{tang. of } \tfrac{1}{4}.$$

$$\frac{\sqrt{1^2+.1989123685^2}-1}{.1989123685}=0984914100=\text{tang. of }\tfrac{1}{8}.$$

$$\frac{\sqrt{1^2+.0984914100^2}-1}{.0984914100}=.0491268530=\text{tang. of }\tfrac{1}{16}.$$

$$\frac{\sqrt{1^2+.0491268530^2}-1}{.0491268530}=.0245486221=\text{tang. of }\tfrac{1}{32}.$$

$$\frac{\sqrt{1^2+.0245486221^2}-1}{.0245486221}=.0122725009=\text{tang. of }\tfrac{1}{64}.$$

$$\frac{\sqrt{1^2+.0122725009^2}-1}{.0122725009}=.0061360222=\text{tang. of }\tfrac{1}{128}.$$

$$\frac{\sqrt{1^2+.0061360222^2}-1}{.0061360222}=.0030679817=\text{tang. of }\tfrac{1}{256}.$$

$$\frac{\sqrt{1^2+.0030679817^2}-1}{.0030679817}=.0015339872=\text{tang. of }\tfrac{1}{512}.$$

$$\frac{\sqrt{1^2+.0015339872^2}-1}{.0015339872}=.0007669931=\text{tang. of }\tfrac{1}{1024}.$$

$$\frac{\sqrt{1^2+.0007669931^2}-1}{.0007669931}=.0003834964=\text{tang. of }\tfrac{1}{2048}.$$

$$\frac{\sqrt{1^2+.0003834964^2}-1}{.0003834964}=.0001917481=\text{tang. of }\tfrac{1}{4096}.$$

$$\frac{\sqrt{1^2+.0001917481^2}-1}{.0001917481}=.0000958740=\text{tang. of }\tfrac{1}{8192}.$$

$$\frac{\sqrt{1^2+.0000958740^2}-1}{.0000958740}=.0000479369=\text{tang. of }\tfrac{1}{16384}\text{ the}$$

arc A B.: and, since the tangent of a very small arc is equal to the arc itself, we have $.0000479369\times16384\times8=6.2831853568$ the circumference when the diameter is 2: hence, by Rule 1st. above, 2 : 6.2831853568 : : 1 : 3.1415926784 the circumference required, which is true to the 7th place of decimals.

For other similar approximations by bisecting an arc, &c., see Simpson's Geometry, page 137, and Hutton's Course of Mathematics, vol. 2, page 34; also by the method of infinite series, see most writers on Fluxions.

PROBLEM 8.

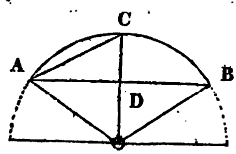
To find the Length of any Circular Arc.

RULE I.

As 360 is to the number of degrees in the arc, so is the whole circumference to the length of the arc.

Note.—The number of degrees in the arc is found by Trigonometry thus:—

Suppose A C B the required arc, whose length is to be found, the radius A O = 10 and chord A B = 16 chains, then 10 (O A) : 8 (A D) : : 1 : $\frac{8}{10}$ = .8 the sine of the angle A O D to radius 1; the degrees answering to this sine are 53.130105, the double of which is 106.260210 the degrees in the arc A C B. But (by Rule 1st. Problem 7.) 3.14159 &c. × 20 = 62.83180 is the circumference of the whole circle. Hence, then, by the above Rule 360 : 106.260210 : : 62.8318 : 18.5458 chains, the length of the required arc A C B.



RULE 2.

Divide the difference between the chord of the whole arc, and eight times the chord of half the arc, by 3, and the quotient will be the length of the arc nearly.—*This is Hutton's 4th Rule.*

EXAMPLE.

By taking again the same numbers as in the last example we have $OD = \sqrt{OA^2 - AD^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$ and $OA - DO = 10 - 6 = 4 = DC$ and the $\sqrt{DC^2 + AD^2} = \sqrt{4^2 + 8^2} = \sqrt{80} = 8.94427 = AC$, then by the above rule $\frac{8.94427 \times 8 - 16}{3} = 18.51805$ chains, the length of the arc nearly.

PROBLEM 9.

To find the Area of a Circle.

RULE 1st.

Multiply half the Circumference by half the Diameter, and the product is the area.

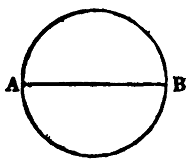
RULE 2.

Multiply the square of the diameter by .7854 and the product is the area.

EXAMPLE.

Required the area of the circle whose diameter A B is 600 links.

(By Prob. 7th,) $600 \times 3.1416 = 1885$ the circumference nearly.



Then, by Rule I.

$$
\begin{array}{r}
2)1885 \\
\hline
942.5 \\
300 \\
\hline
2.827500 \\
4 \\
\hline
3.310000 \\
40 \\
\hline
12.40000 \\
\hline
\end{array}
$$

Ans. 2A. 3R. 12P.

By Rule 2.

	.7854
	360000
600	471240000
600	23562
360000	2.827440000
	4
	3.30976
	40
	12.39040

Ans. 2A. 3R. 12P.

Note.—Besides the above methods, Dr. Hutton, in his Mensuration, page 493, has approximated the area of a quadrant (whose radius is 1,) very near the truth, by means of equidistant ordinates, and considering that part a parabola which lays between the curve and the second least ordinate. Now the same may be found as near the truth without introducing the parabola; for (by Prop. 1, Simpson's Dissertations, page 109,) if the co-sine of 60° (radius 1,) be divided into 10 equal parts by 11 ordinates, the first or least of which will be the sine of 60°, and the last or greatest equal to the radius, all the intermediate ones will be the sines of arcs, whose versed sines are .55, .60, .65, .70, .75, .80, .85, .90., .95. Now it is very well known that the co-sine of 60° is equal to half radius, consequently by the property of the circle we have

$\sqrt{1.5 \times .5} = \sqrt{.75} = .8660254$ the first ordinate.
$\sqrt{1.45 \times .55} = \sqrt{.7975} = .8930285$ the second.
$\sqrt{1.40 \times .60} = \sqrt{.8400} = .9165151$ the third.
$\sqrt{1.35 \times .65} = \sqrt{.8775} = .9367496$ the fourth.
$\sqrt{1.30 \times .70} = \sqrt{.9100} = .9539392$ the fifth.
$\sqrt{1.25 \times .75} = \sqrt{.9375} = .9682458$ the sixth.
$\sqrt{1.20 \times .80} = \sqrt{.9600} = .9797958$ the seventh.
$\sqrt{1.15 \times .85} = \sqrt{.9775} = .9886859$ the eighth.
$\sqrt{1.10 \times .90} = \sqrt{.9900} = .9949874$ the ninth.
$\sqrt{1.05 \times .95} = \sqrt{.9975} = .9987492$ the tenth.

And radius $= 1.000000$ the eleventh, as mentioned above. Now the area, by Hutton's rule, laid down page 491,

(or Simpson's page 110, which is the same,) is equal $(A+4B+2C)\times\frac{1}{3}D$, where $A =$ the sum of the first and last ordinates; $B =$ the sum of the even, or second, fourth, sixth, &c. ordinates; $C =$ the sum of all the rest; and $D =$ the common distance between the ordinates. Hence we have $A = 1. + .8660254 = 1.8660254$; $B = .8930285 + .9367496 + .9682458 + .9886859 + .9987492 = 4.7854590$; $C = .9165151 + .9539392 + .9797958 + .9949874 = 3.8452375$; and $\frac{1}{3}D = \frac{.05}{3}$ and consequently $(A+4B+2C)\times\frac{1}{3}D = .47830560$, the area; from which deducting the area of the right angled triangle, whose base is the co-sine divided, and perpendicular its corresponding sine or first ordinate $= \frac{.8660254\times .5}{2} = .21650635$, we have $.47830560 - .21650635 = .26179925$, the area of a sector of 30°. Hence $.26179925 \times 3 = .78539775$ the area of the quadrant, which is very near the truth.

Note 2.—If the co-sine be divided into 20 equal parts, the area of the quadrant will be .78539808, which is a little nearer.

Note 3.—By applying the rule laid down in section 3, for equidistant offsets, when the co-sine is divided into 20 equal parts, you will have the area of the quadrant in that case $= .7853078$, which is very near likewise.

PROBLEM 10.

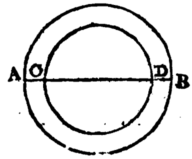
To find the Area included between two concentric Circles.

RULE.

The sum of the diameters multiplied by the difference of the diameters, and the product again by, .7854 will be the area.

EXAMPLE.

What is the area included between the two concentric circles, whose diameters A B and C D are 6 and 4 chains?



```
 600
 400
────
1000 = sum of the diameters.
 200 = difference of the diameters.
──────
200000
──────

      .7854
     200000
     ──────
1.5708000000
         4
     ──────
     2.2832
         40
     ──────
    11.3280
     ──────       Ans. 1A. 2R. 11P.
```

PROBLEM 11.

To find the Area of a Sector of a Circle.

RULE I.

Multiply the radius of the circle by half the arc of the sector, and the product will be the area.—*(This is Hutton's 1st Rule.)*

EXAMPLE.

What is the area of the sector A O B C A, whose radius A O is 10, and chord A B of its arc = 16 chains ?—(See Fig. to Prob. 8.)

By Problem 8, the length of the arc A C B is 18.545887, consequently (by the above rule,) half the arc A C B × A O = 9.272943 × 10 = 92.72943 chains, the area required.

RULE II.

As 360 is to the whole area of the circle, so are the degrees in the arc of the sector to its area.—*(This is Hutton's 2nd Rule.)*

EXAMPLE.

What is the area of a sector, whose radius and chord of its arc, are the same as in the last example?

By Problem 7, the degrees in the arc are 106.26021. Also by Problem 9, $20 \times 20 \times .7854 = 314.16$, the area of the whole circle. Then, (by the above rule,) 360 : 314.16 : : 106.26021 : 92.7297 chains, the same as before nearly.

PROBLEM 12.

To find the Area of the Segment of a Circle.

RULE I.

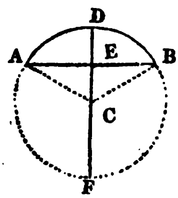
First find the area of the sector, having the same arc as the required segment.

Next find the area of the triangle, formed by the two radii and chord of the segment.

Then the sum of these areas is equal the area of the greater segment, or their difference to that of the lesser.—*(This is Hutton's 1st Rule.)*

EXAMPLE.

Required the area of the segment A D B A, whose chord A B is 16, and its versed sine or height D E $=$ 4 chains.



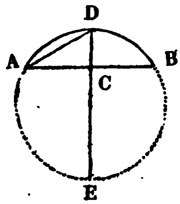
1st.—The area of the sector by the last example is 92.7297, and the area of the triangle A B C (by Prob. 2,) $= \frac{AB + EC}{2} = \frac{16 \times 6}{2} = 48$. Then (by the above rule,) $92.7297 - 48 = 44.7297$ chains, the area of the segment A D B A, which was required.

RULE II.

To the chord of the whole arc, add $\frac{4}{3}$ of the chord of half the arc; multiply the sum by the versed sine, or height of the segment, and $\frac{4}{10}$ of the product will be the area of the segment nearly.—*(This is Hutton's 5th Rule.)*

EXAMPLE.

Suppose the chord A B, and the versed sine or height C D of the segment A D B C A the same as before, required the area.



Here $AD = \sqrt{AC^2 + DC^2} = \sqrt{8^2 + 4^2} = \sqrt{80} = 8.94427$; then $(AB + \frac{4}{3}\,AD) \times CD \times \frac{4}{10} = \overline{16 + \frac{4}{3} \times 8.94427} \times 4 \times \frac{4}{10} = 44.681104$, the area of the segment A D B C A nearly.

Note.—The readiest method of finding the area of a segment of a circle, is by the table of circular segments laid down at the latter end of Hutton's Mensuration, which is as follows:—

RULE III.

Divide the versed sine, or height, by the diameter, and the quotient will be the tabular versed sine; then multiply the segment corresponding to it, by the square of the diameter, and it will be the area.—*(This is Hutton's 9th Rule.)*

EXAMPLE.

Supposing the dimensions the same as in the last example, we have $DC \div ED = \frac{4}{20} = .2$ the tabular versed sine. The segment corresponding to it is .11182380. Hence

(by the above rule,) $.11182380 \times 20 \times 20 = 44{,}72952$ chains, the area, which is very near the same as that given by the first rule.

PROBLEM 13.

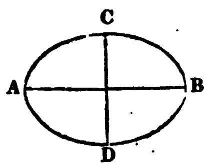
To find the Circumference of an Ellipse.

RULE.

Multiply the sum of the two axes, by the half of 3.1416, and the product will be the circumference nearly.—*(This is Hutton's 2nd Rule.)*

EXAMPLE.

Required the circumference of an ellipse, whose two axes A B, D C are 30 and 20 chains respectively.



By the rule $(AB + CD) \times \frac{3.1416}{2} = (30 + 20) \times \frac{3.1416}{2}$ $= 78.5400$ chains, the circumference nearly.

RULE.

Multiply the square root of half the sum of the squares of the two axes by 3.1416, and the product will be the circumference nearly.—*(This is Hutton's 3rd Rule.)*

EXAMPLE.

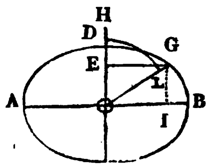
Every thing remaining the same as in the last example, we have $\frac{\sqrt{AB^2 + CD^2}}{2} \times 3.14159 = \frac{\sqrt{30^2 + 20^2}}{2} \times 3.14159$ $= 80.09537$ chains the circumference nearly.

PROBLEM 14.

To find the length of an Arc D G of an Ellipse.

RULE

Find the length of the circular arc H L, intercepted by O G, OD, whose radius is half the sum of the said lines O G, O D, and it will be equal to the elliptic arc nearly.— *(This is Hutton's 2nd Rule.)*



EXAMPLE.

Required the length of the arc D G, the two axes A B, D F, being 30 and 20, and the distance O I, 10 chains respectively.

First by the property of the ellipse A O^2 : D O^2 : : A I $\times$ B I : G I^2 $= \dfrac{10^2 \times 25 \times 5}{15 \times 15} = 55.555$, and (by Euclid 47.1, O G $= \sqrt{OI^2 + GI^2} = \sqrt{100 + 55.555} = 12.47219$. Then $\frac{1}{2}$ O D $+ \frac{1}{2}$ O G $= \dfrac{10 + 12.47219}{2} = 11.23609 =$ O L, the radius of the circular arc H L; but O I $\div$ O G $= \dfrac{10}{12.47219} = .801783$ is the sine of the angle E O G, answering to 53.300709 degrees. Also (by Rule 1, Prob. 7,) $12.47219 \times 2 \times 3.1416 = 78.365264$, the circumference of a circle, whose radius is O G. Then (by Rule to Prob. 8,) as 360 : 78.365264 : : 53.300709 : 11.602567, the circular arc H L, or elliptic arc D G, nearly.

PROBLEM 15.

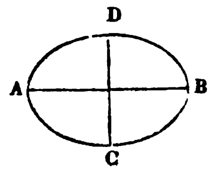
To find the Area of an Ellipse.

RULE.

Multiply continually together the two axes; and the number .7854 for the area.

EXAMPLES.

Required the area of the ellipse A B C D, whose two axes A B and D C are 4 and 3 chains respectively.



$$\begin{array}{r} .7854 \\ 300 \\ \hline 235.6200 \\ 400 \\ \hline 94248.0000 \\ 4 \\ \hline 3.76992 \\ 40 \\ \hline 30.79680 \\ \hline \end{array}$$

Ans. 0A. 3R. 30P.

PROBLEM 16.

To find the Area of an Elliptic Segment, cut off by a Double Ordinate or Line perpendicular to either Axe.

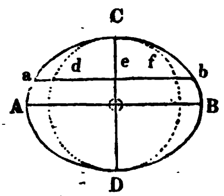
RULE.

Find the area of the corresponding circular segment, described upon the same axe to which the base of the segment

is perpendicular. Then, by the rule of proportion, say, as that axe, which is the diameter of the circle, is to the other axe; so is the area of the circular segment, to the area of the elliptic segment.—*(This is Hutton's 1st Rule.)*

EXAMPLE.

Required the area of the elliptic segment a c b a, cut off by the line a b; when A B = 30, C D = 20, and C e = 4 chains.



With radius $\frac{1}{2}$ C D, describe the circle C f D d C: Then O e being = 6, we have (by Prob. 12.) the area of the circular segment d C f d = 44.7297; consequently, by the above rule, C D = 20 : A B = 30 : : segment d C f d = 44.7297 : segment a C b a = 67.39455 chains, as was required.

PROBLEM 17.

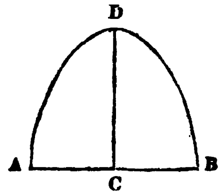
To find the Area of a Parabola.

RULE.

Multiply the base or ordinate by the abscissa or height, and $\frac{2}{3}$ the product will be the area.

EXAMPLE.

Required the area of the parabola A B D, whose abscissa and ordinate D C and A B are 3 and 5 chains respectively.

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     500
     300
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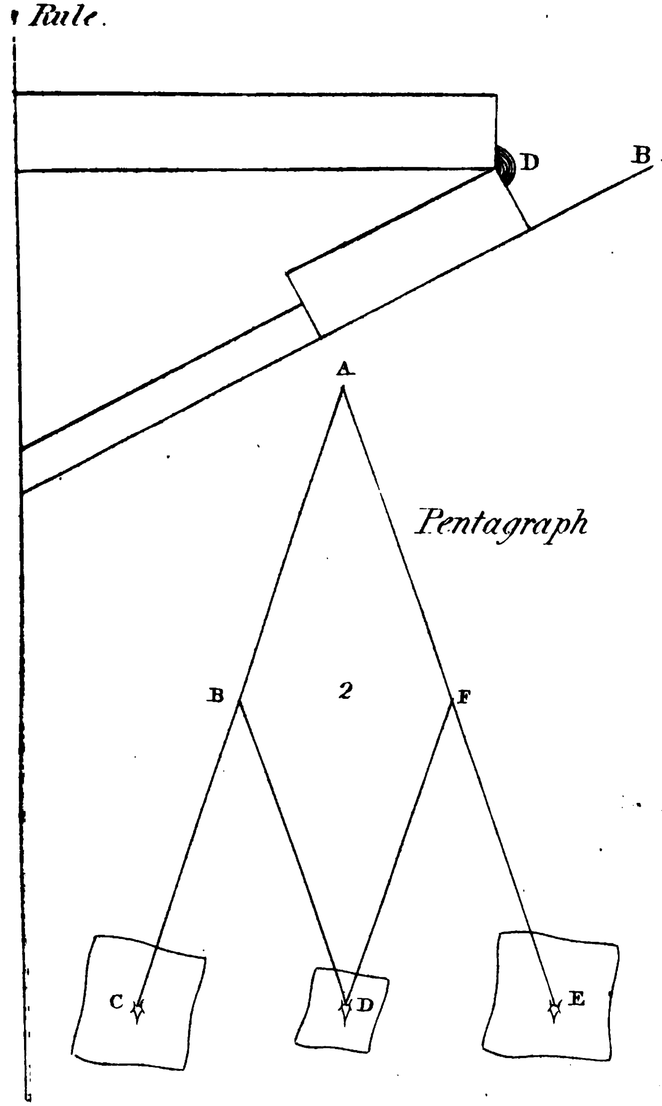
Ans. 1A. 0R. 1P.

For a clear demonstration of all the rules given in this section, from first principles, see Dr. Hutton's large Mensuration, where various other rules are laid down and demonstrated likewise, that may be applied to the solution of different Problems, respecting areas.

Plate 1



LAND SURVEYING.

PART II.

SECTION I.

OF LAND SURVEYING, &c.

By Land Surveying is meant the taking of the dimensions of a quantity of land, in order to ascertain the contents, or make a plan of it, by the problems of common Geometry. Hence, having given a few of the most useful definitions and problems, in practical Geometry, which it is requisite for every Surveyor to know, I shall next proceed to point out the most useful Instruments employed in surveying, and also the marks and characters used in the field-book, before I enter upon the method of taking dimensions, &c.

USE AND DESCRIPTION OF THE INSTRUMENTS.

I. THE CHAIN.

"Gunter's chain" is the chief instrument used in surveying, for by it all the distances are measured in the field, except the short offsets. It is 22 yards in length, and

divided into 100 equal parts, called links. Each link is equal to 7.92 inches; and 1000 of them in length, and 100 in breadth (equal to 10 chains in length, and one in breadth) are one acre statute measure. The end links are made into bows adapted to the hand; and at the end of each tenth link, is affixed a small plate of brass serrated, or notched into points descriptive of its distance from the end. Thus at the end of the tenth link from each end, the brass has one point; at the twentieth, two points; at the thirtieth, three points, &c.; and at the fiftieth, the brass is a circle.

2. THE ARROWS.

The arrows are used in determining the lengths of lines, measured by the chain, by sticking one into the ground at the end of every chain; and counting them at the end of the line. They are ten in number, and made of iron wire or wood, about 12 inches long, with one end sharp, and the other end (if made of wire,) bent round in the form of a ring; so that the leader and follower of the chain, when measuring, may, for convenience, hang them on their fingers.

3. THE OFFSET STAFF.

The offset-staff is used in measuring the offsets, or perpendicular distances, between the chain line and bends in the hedge, &c. It is a small fir pole, from 10 to 15 links in length, which are graduated and numbered upon it.

4. THE POLES WITH FLAGS.

The poles with flags, are used both to point out the line of direction, and to fix in or near the corner of a field, when surveying a single field, or in the stations, &c. when surveying a large estate, to assist the surveyor in his operations. They are made of light wood, like the offset-staff, but longer, with a flag at the top, and a picket at the bottom of each.

5. THE BEAM COMPASSES.

The beam compasses are used in describing arcs with a greater radius than can be performed by the other compasses. They are composed of a small beam of wood or metal, about a yard long, and of two other pieces about three inches long, with a sharp point at one end, and a hole perforated through the middle of each, of the same dimensions as the end of the beam, so that they may be slided on it, to any distance required, one from another.

Note.—Instead of the beam compasses, some surveyors use a long narrow piece of paper, and two pins—a method which may answer very well if applied with care.

6. THE PLOTTING SCALES.

The plotting scales are used in laying down, on the plan, the lines or distances measured in the field by the chain, &c. They are made of brass or ivory, &c., 12 inches long, and divided into chains and tenths. Those most used by surveyors are of 3 and 4 chains to an inch, graduated on the edge; so that, in planning, any offset can be marked off, without using a pair of compasses. These scales are made exceeding thin at the edges, for the purpose of setting off distances correctly; and having, at zero, a line, perpendicular to the edges, drawn quite across, perpendiculars can be raised by them with ease and accuracy.

7. THE HORN RULER.

The horn ruler is used in reducing crooked fences to straight ones. It is made of a thin piece of horn, so that, when laid upon any crooked line, the said line may be perfectly distinguished through it; and, consequently, by drawing a fine line by its edge, a quantity may be taken in, equal to a quantity left out, &c.

8. THE PARALLEL RULER.

The parallel ruler is used in reducing an irregular figure to a trapezium or triangle. It is formed of two equal rulers, connected together by two small bars, (*see figure* 3, *plate* 1,) at a small parallel distance from each other, and moveable about the points which connect them.

9. THE LEVELLING RULE.

The levelling rule is used in reducing hills, when measuring, to an horizontal level. It is formed of three short rulers, A B C, (*figure* 1, *plate* 1,) two of which are of equal length, viz. A and B, and are called radii; the other (C) being shorter, is called the chord. The two radii are fixed together with a joint at D; the chord is likewise fixed into the other end of A, in the same manner, after cutting part of it away; so that the remaining part and chord, when folded together, may be equal to the other radius. The inside of the chord that shuts against the radius, is graduated, and shows how many links the chain must be pulled on, in every chain's length, when measuring up or down a hill, to reduce it to an horizontal level.

Note.—The distance on the chord from the joint, with the breadth of the piece cut off the radius, (or breadth of the chord) is similar to the chord of that angle whose secant exceeds the radius or base by 1, 2, 3, &c. links; consequently, having given the base and hypothenuse of a right angled triangle, the angle which any hill makes with the horizon is (by Trig.) readily found. Thus,

As 101, 102, &c. : 1 : : 100 : 99009, 98039, &c. the natural co-sine of the angle included, by the base and hypothenuse of the hill.

Hence is derived the following table of angles, when the chain has to be drawn forwards 1, 2, 3, &c. links, to reduce the hill to a horizontal level.

Deg.	*Min.*	*Links.*	*Deg.*	*Min.*	*Links.*
8	.. 4	1	37	.. 28	26
11	.. 22	2	38	.. 3	27
13	.. 52	3	38	.. 38	28
15	.. 57	4	39	.. 11	29
17	.. 45	5	39	.. 43	30
19	.. 22	6	40	.. 14	31
20	.. 50	7	40	.. 45	32
22	.. 12	8	41	.. 15	33
23	.. 27	9	41	.. 44	34
24	.. 37	10	42	.. 12	35
25	.. 43	11	42	.. 40	36
26	.. 46	12	43	.. 7	37
27	.. 45	13	43	.. 34	38
28	.. 42	14	43	.. 59	39
29	.. 35	15	44	.. 25	40
30	.. 27	16	44	.. 50	41
31	.. 16	17	45	.. 14	42
32	.. 4	18	45	.. 38	43
32	.. 49	19	46	.. 1	44
33	.. 33	20	46	.. 24	45
34	.. 16	21	46	.. 46	46
34	.. 57	22	47	.. 8	47
35	.. 37	23	47	.. 30	48
36	.. 15	24	47	.. 51	49
36	.. 52	25	48	.. 11	50

10. OF THE PENTAGRAPH.

The pentagraph is used in copying one plan from another, and may be employed in copying them to any size whatever. It consists of four rulers, two of which are about 18 inches long, the other two half that length; the two short ones, as well as the two long ones, are joined together by their ends; the other ends of the short ones are fixed by two joints into the middle of the long rulers, so that the distance from the centres of the said joints to the centres of the joints that connect the ends of both the long and short rulers together, as also to the centres of the holes in the other ends of the long rulers, (wherein are fixed proper instruments for tracing the

lines, &c.) shall be equal to each other; forming always, when rightly put together, the figure of a parallelogram.

For a further description, &c. see Dr. Hutton's Mathematical and Philosophical Dictionary; where may be seen likewise, a further description, &c. of most of the other instruments mentioned above.

Note.—Besides the instruments here mentioned, it is necessary for every surveyor to be provided with a case of small instruments (i. e.) compasses, steel pens, prickers, &c.; also a quantity of black lead pencils, Indian rubber, Indian ink, &c. &c.

OF THE USE AND DESCRIPTION OF THE FIELD BOOK.

In the field book are entered all the remarks, &c. noticed in surveying, viz. the lengths of lines; the distance between one station and another; the lengths of offsets; the distance of each offset from the beginning of the chain line; the hedges passed and gone through; the houses passed; with the roads, woods, waters, &c. It is comprised of blank leaves of paper, with each page ruled into columns for the remarks, which are always entered, by writing up the leaf instead of down.—*(See field book, plate* 1.*)*

In the first column are the stations or trigons you go from; in the second, the offsets taken to the left hand of the chain line; in the third, the distances measured, from whence offsets are taken, stations made, &c.; in the fourth, the offsets taken to the right hand; in the fifth, the marks and characters, stations, &c. made in measuring the lines; also the stations or trigons arrived at, as well as the sketching of part of the hedges, sewers, roads, houses, &c. crossed or passed; in the sixth, likewise, part of the sketching; so that by observing well the book, every separate crank in the fence, house, road, spring, &c. is both speedily and accurately laid down on the plan.

Note.—The line whereon, or along which, are sketched the fences, houses, &c. is supposed to be the chain line, or line measured by the chain; having the fences, houses, roads, &c. marked upon or near it, opposite their respective distances (from the station measured from) in the third column, as the line A B opposite; which method of sketching, in my opinion, is much better than sketching through the figures, as practised by many surveyors.

OF THE MARKS AND CHARACTERS USED IN THE FIELD BOOK.

Marks and Characters.	
⊙	Signifies a square hole cut out of the ground, wherein is put a piece of chip, with a number upon it, and it is called a station. *Note.*—Instead of numbers, some surveyors use the alphabet.
△	A triangular hole, cut out of the ground, in a line measured before, and is called a *trigon.* It is always entered into the book, so many links East, West, North, or South of the nearest station in the said line, (as in the field book, plate 1,) where the first line terminates in a trigon 120 links South of ⊙ 1, and the second begins at 20 links West of ⊙ 6.
+ & —	More and less, as in algebra. Hence, when a crank in a fence, &c. lays at right angles to the chain line, the length of the said crank is put after the other offset, with the sign + or — between them, as at 4 chains, where the offset is put down 20 + 10; the length of the crank *lm* being 10 links, and the distance between the chain line and *l* 20. Also, at 6 chains, where it is put down 30—10, by reason of the distance between the chain line and angle *h* being 30, and the crank *gh* equal to 10 links.
÷	Division, or a hedge, &c.
> or ∠	An angle, or corner of a field, &c.
×	Not multiplication; but it is called in the field book a cross.
Ent. H.	That at one chain, an offset of 20 links was taken to the first corner, or entering of a house.

Marks and Characters.

Left H. That at 125 links, the said house was left.

Note.—Sometimes one house is left, and another entered, at the same time, in which case it is booked thus—left and entered H.

Pas ÷ *st.* That at two chains, an offset was taken of 20 links to the hedge-end *(ik)* which hedge is taken as a straight one, and is laid down upon the plan without tracing, after the other end is determined.

Note.—It is always to be undertood when *(st.)* stands after ÷ that the fence is a straight one.

Pas ∓ That at 5 chains, an offset was taken to the hedge-end *(ik)* of 30 links, in the direction of the said hedge.

× ÷ *st* to ∟*e*. That at 8 chains, the fence was crossed at right angles, and an offset taken on each side to the angles *e* and *f*.

Ent. Rd. That at 9 chains a road was entered.

Left Rd. That at 9 chains 50 links the said road was left.

Note.—When the road is not a lane, it is denoted by a double dotted line.

Pass ÷ That at 10 chains, the hedge-end *(cd)* was passed; but is not a straight one.

× ÷ or + ÷ That at 20 chains, the hedge was crossed in an oblique direction.

It also signifies the same in any other position.

A tree or fence is denoted by a small figure of a tree, turned to or from the chain line, according as the tree or fence belongs to the ground the chain is then in, or to the adjoining.

× W. That at 21 chains, a water was crossed.

Spring. That at 25 chains, an offset of 10 links was taken to a spring of water.

Marks and Characters.

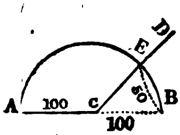
× path. That at 30 chains, a foot-path was crossed.

Note.—A foot-path is always denoted by a single dotted line.

Turn'd scale. That the number of chains measured, is equal the number of chains graduated, on the scale you intend to plot from. As 36 is the number on a 12-inch scale, of 3 chains to an inch; and 48, at 4 chains to an inch.

And when a line of 36 or 48 chains is measured, a double line is drawn across the 2nd, 3rd, and 4th columns; the leader also takes all the arrows, and every thing is noted down, as if beginning with a new line.

Note.—A single line is drawn across the 2nd, 3rd, and 4th column after every station that is made, and entered between the extremities of the two ends. A single line is likewise drawn across all the columns at the end of every line; also a double line at the finishing of every triangle or trapezium constructed on the base line.



That the obtuse angle A C D is taken with the chain; and that A C = 100, is part of the line last measured, C B = 100, the part produced, E B = 50, the chord of the supplement of the said angle, and C D the line we are next going to measure.

Left off. That the offsets are discontinued, or part of a fence is left untraced.

Begun. That the offsets are begun with again, and that the fence or space from whence we left off, has to be traced by lines running nearer to it, at some future time.

SECTION II.

OF PRACTICAL PROBLEMS IN SURVEYING.

By practical problems, are meant, such problems as are practised in surveying, *viz.* such as measuring a base or straight line with the chain, taking an angle, laying them down upon the plan, reducing crooked fences to straight ones by the horn ruler, reducing regular or irregular fields to trapeziums or triangles, by the parallel ruler, &c. &c.

PROBLEM I.

To measure the length of a Base, or any other Straight Line, with the Chain, and take the Offsets, &c. that fall thereon.

RULE OR DIRECTIONS.

Having selected a visible object at a distance, in the direction of which to measure, let the leader of the chain take all the ten arrows, holding one in that hand which pulls the chain, and advance forward one chain from the follower, who stands with his end of it at ⊙ 1, to direct how and when the first arrow shall be put down, in a line with the object, in the following manner:—After the leader has advanced one chain, let him stand with his side to the follower, or his face towards the chain, holding that hand straight from him that holds the bow of the chain and single arrow, with the arrow point upon the surface of the ground; let the follower then direct him to put his arrow to or from him, until it comes in a direct line with the follower's station, and the distant object, before it is stuck into the ground, by waving his hand, or by repeating

the word backwards or forwards. Having thus got the leader to put down his first arrow, let the chain lie upon the ground until the offsets, their distances on the chain line, the distances of the new stations, &c. if any occur in that chain's length, are booked; after that, let him take up the chain, and advance again, with another arrow in the hand he draws it with, until the follower arrives at the first arrow put into the ground, where he must hold the bow close to the arrow, and direct the leader when and where to put down his next or second arrow, as before. And thus, must they measure and advance, chain after chain, taking the offsets, &c. wherever they occur, until they arrive at the end of the line, the length of which, is the number of arrows the follower holds in his hand, added to the number of links between the arrow last put down, and the end of the line, and is noted down into the book in links, as follows, *viz.*—If the follower holds eight arrows, and the distance between the last arrow and the end of the line be ten links, the length of the line is then booked in the third column 810.

Note.—In measuring any line, the sketching part is always done after you have taken and entered every separate offset, and not before,—as in measuring from ⊙ 20.—(*See field book, plate* 1.) After entering into your field book its length (20), draw a short line, as the line *x y*, at any distance you think will best suit your purpose, from the chain line AB. Then take your next, to the first corner of the house, and enter it likewise; after that, bring up your sketching to the house, and describe its side AB. After you have taken and entered your third, at leaving the house, bring up your sketching again, which is the end of the house, and draw the other side, noting down its length, (80): thus observing always to bring up the sketching just opposite the place where you have entered your offset.

Note 2.—When the number of chains in the line exceeds the number of arrows, or 10, which is frequently the case, let the follower, after he has got all the ten arrows, and directed the leader where to lay down the eleventh chain in a line with the object; count them over, and deliver nine of them to the leader, sticking the other himself into the ground, at the leader's end of the chain. Next, let the leader count the arrows given him, for fear of a mistake, and then advance forwards in the same direction, guided by the follower, until he disposes of them all as before; observing always to change

and count the arrows as above directed, until the line be finished; the length of which, will be the number of tens added to the arrows the follower holds in his hand—plus the number of links between him and the end of the line, and is booked as before.

And in measuring any line, especially a long one, it is always to be observed, that the leader of the chain, after he has put down his first arrow according to the directions of the follower, should stand at the said arrow, and try if he can find any object at a distance, in a line with him and the follower, when standing at the station from which he is going. If he can, he then will be able always to keep himself in the line with which he begun, providing the follower should lose *his object*. And the same should be done in any part of the line, provided he be likely to lose his first object.

Note 3.—When you have measured the whole length of any line, or until you think it convenient to end it, and measure in another direction, observe that the follower of the chain, (after making a ⊙ or ▽ at the end of the line, and entering its length into your book,) delivers all his arrows to the leader, and that the leader counts them over before he begins to measure a new line.

Note 4.—Observe always, when you have occasion to take both an offset and make a ⊙ in one chain's length, to enter them in your book as they come in course: *i. e.* if it be necessary, an offset should be taken at 10 links from the follower, and a station made at 20, take the offset first, and enter it into your book, afterwards make the ⊙ &c.; or, if the ⊙ should be made at 10 links, and the offset taken at 20, then make the ⊙ first, and enter it in your book; afterwards take the offset, &c.; for by so doing, the book is kept clear of difficulties, and, consequently, the line is very readily laid down on the plan.

It is also to be observed, when you want to take an offset from the chain to a fence, &c. that you stand at the link upon which the perpendicular falls, and enter its distance from the beginning of the line, into your field book first; after that, measure the offset, and enter it likewise; and before you leave the fence, &c. mark where another will fall, which place keep in your eye, until you come opposite it, &c., and so proceed to the end of the line.

Note 5.—When you have a very long line to measure, choose two or three visible objects, if possible, in the line of direction; for then you may readily know whether you have deviated or not, from the line you first begun to measure, by observing, when standing at an arrow, whether or not, the objects and you are in one direct straight line.

Note 6.—When you have to cross a hill or valley, with any line, observe always that you reduce the hill to a horizontal level, as near as you can, by your levelling rule, or some other method; otherwise it will not answer its position on the plan; for, it is to be understood, through the whole of this work, that every line measured and entered in the field book, is on the supposition that the surface of the ground measured, is either horizontal or

reduced to that state; and that the hypothenusal measure, or that of the real surface, is not regarded, being of no use where the plan of an estate is required. I shall not advance any thing respecting this subject, in the present work, on account of its having been so long a controverted point among surveyors; but I will candidly tell my readers, that if they only understand what is here laid down respecting the horizontal measure, they will be competent judges, how and in what manner to proceed with the hypothenusal, should it fall in their way. For the supremacy of the two methods, *see Dr. Hutton's Montucla's Translation of Ozanam, vol.* 1, *page* 291.

Before you begin to survey any large estate, you must measure carefully the length of your chain, when stretched out upon a piece of horizontal ground, and mark the length you intend to have it, by driving a wooden peg fast into the ground at each end. This length or distance, between the two pegs, must form your standard during the survey, and you must daily make your chain equal to it; otherwise, in time, by stretching, &c. it will be over long, and consequently, variable—and this variation, from the standard or length you first begun with, will make the whole of the work incorrect.

In measuring a large survey, the chain should be about two inches more than the statute length, otherwise, by the unevenness of the earth's surface, crossing of fences, and suspending the chain from the top of the arrows, when measuring, you will not give true statute measure.

PROBLEM 2.

To take an Angle with the Chain.

DIRECTIONS FOR AN OBTUSE ANGLE.

After you have measured your first loose line, as far as you think necessary, make a ⊙, from which measure in the direction you wish to proceed with your second line. But observe to stop, and not measure further from the ⊙ than you can produce your first line. At the place where you stop, leave a mark. Then produce the first line, until the part produced beyond the ⊙ be equal to the length of the part measured of the second. Next measure the distance between their extremities; afterwards proceed with the second line, and it is done.

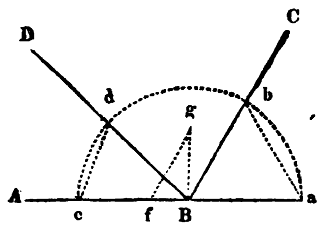
Cor.—Hence, before the ⊙ is made in the first line, by leaving a mark at any known distance from it, and mea-

suring from the same distance in the second to the said mark, an acute angle is likewise determined.

Note.—By taking the distance between the two extremities very exactly, the angle may be pretty accurately laid down upon the plan, as will be shewn hereafter.

EXAMPLE.

Let it be required to take the angle A B C.



First, by measuring A B and making B a = B b, then by measuring a b the distance between the two extreme points, the position of the line B C becomes known, and consequently the angle A B C determined by the chain.

And in the same manner, by making B d = B c, (according to the Cor.) is the angle A B D determined.

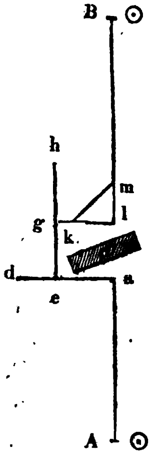
EXAMPLE 2.

Let it be required to take the right angle A B g with the chain.

After measuring the line A B, take B f equal to 30 links. Fix one end of the chain fast at f, the 90th link at B, then take hold of the 50th link, and stretch the chain upon the

ground, and it will form the other two sides f g, B g of the right angled triangle f B g, consequently the angle A B g is determined.

This triangle has its sides in arithmetical progression, and was made use of by Ozanam, in erecting a perpendicular at the extremity of a given line.—*(See Hutton's Translation, vol. 1, part 2, prob. 1, page 262.)*



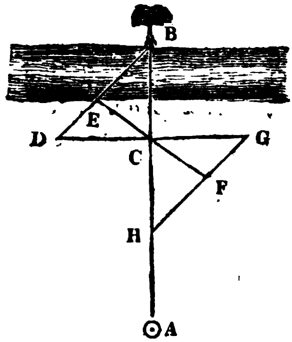
If it should happen, in measuring any line, (as the line A B,) that a building should obstruct the view of the object, in the direction of which you are measuring, measure as near to the building (x) as you think will be most convenient for erecting a perpendicular upon the line, as to the point a. Then, by the last example, or with your offset-staff, as will be shewn hereafter, erect the perpendicular a d. Measure from a towards d, until you can see to measure past the building, a line parallel to A B, as to e. At e erect the perpendicular e h, and measure therefrom to g. From g let fall the perpendicular g l. Next g l measure, and if the perpendiculars be rightly erected, &c. you will have g l = a e; also a l = e g. Hence, from l measure to the end.

If you are measuring only in the direction of one object, after making g l equal to a e, construct the right angled triangle k l m, and if the work be right, l m B will be one direct straight line.

And in the very same manner, you may pass a large pond, or any other impediment, that your lines may fall across, when measuring.

If you have two objects, the building, &c. may be passed by constructing one of the above triangles, and producing the hypothenuse until the perpendicular let fall from l, will intersect it; then find the distance l a by similar triangles.

The above methods appear to me very old, for we find particular cases of them explained in *Hutton's Translation, prob.* 3, *page* 266,—one of which is as follows;—



Suppose, in measuring the line A B, that you are impeded by a river, from getting to the point B, which point you want to be at, in order to fix your line in its true place. Now, after you have got near to the river, as to the point C, fix a pole; fix also another in any place you please, as at D; also another at E, in a straight line with D and B; produce D C and E C beyond the point C, so that you can make C G = D C, and C F = E C; then fix a pole in G and F. Lastly, fix another in the line you have measured, as at H, so as to be in a straight line, both with C B and F G, then will the distance H C be equal C B, which distance measure, and add it to A C, and it is done.

PROBLEM 3.

To find, with the offset staff, the Place upon the chain line where a Perpendicular will fall, from a given point at a distance.

RULE.

Hold your offset staff at right angles across the chain, when laid straight in the line you are measuring, and observe whether its extremity be in a line with you and the point at a distance. If it be, the link crossed by the pole

is the place where the perpendicular will fall. If it is not, move backwards or forwards, as occasion may require, along the chain with your pole in the above angular position, until you determine the place, and it is done.

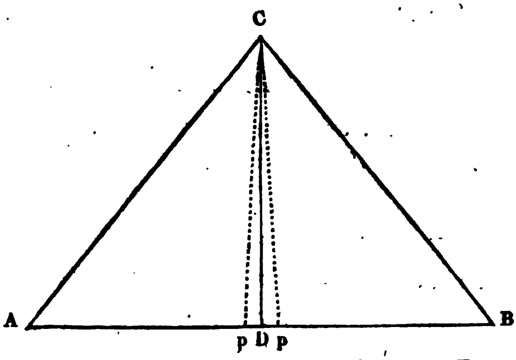
Note.—When the perpendicular exceeds 60 or 70 links, it is the safest way to measure its length with the chain.

REMARK.

The above method of determining the place in the chain line, or base of a triangle, where a perpendicular will fall from a given point, I am quite aware will be objected to as erroneous in principle, by those surveyors who use the *cross*, I shall, therefore, advance a single example, to vindicate its utility, as being both simple and expeditious, and even accurate, when performed by a person thoroughly skilled in the business.

EXAMPLE.

Suppose A B C to represent a triangular field, the base A B of which is 1000, and the true perpendicular C D = 600 links. But in measuring from A towards B, I find, by my method, the point p to be the place where the perpendicular will fall from the point C; and that the perpendicular C p = 601 links. Required the distance p D, and the difference between the true and false area.



First, in order to determine the distance D p, when C p exceeds C D, by one link, we have by Euclid 47.1 $\sqrt{601^2 - 600^2} = \sqrt{1201} = 34.6 =$ D p; and twice D p = p p = 69.2 links, the space on the base, in any part of which the false perpendicular will fall, without exceeding the

true perpendicular more than one link; and to make the perpendicular fall within the above space, by laying a straight pole across a straight line, is, I think, a matter very easy to perform; whilst the difference of the areas, should the false perpendicular even fall at either extremity of the above space, will only be eight tenths of a perch;
$\frac{600 \times 1000}{2} = \frac{600000}{2} = 3$ acres, being the true area, and
$\frac{601 \times 1000}{2} = \frac{601000}{2} = 3$A. 0R. 0.8P. the false area.
Likewise, if we suppose the true perpendicular 1200, and the false 1201 links, by proceeding as above, we have p D = 49 links, and 2 p D = 98; also the difference between the true and false, in that case, only .48 of a perch; which two differences are either of them less than the difference of two separate surveys of the same field would be, provided they were both made and taken by one and the same person. Hence, let the *cross* be as true an instrument as it may, it appears to me very evident, that its use may be dispensed with, in surveying either a single field, or a large estate, provided the surveyor be only furnished with a straight pole.

PROBLEM 4.

To transfer any Line, with the Remarks, &c. made in Measuring it, from the Field Book to the Plan.

DIRECTIONS.

When the line to be transferred is a base line, draw one upon your plan at pleasure, and make ⊙1 upon it, where you think will be most convenient. Then lay the scale you intend to plot from, along the said line, with one of its extremities at ⊙1, and mark off, with your pencil or pricker, all the other ⊙s made thereon, in their respective places. Next, after numbering the stations then made, begin at ⊙1 and mark the places on the chain line from whence the offsets are taken. Lastly, with the scale per-

pendicular to the chain line, lay down the length of each offset in its proper place, and draw a line through the extremity of each, and it is done.

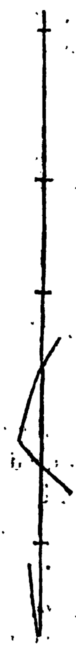
When the line is a fast one, *i. e.* when it extends from one known fixed point, or ⊙, to another, measure the distance between the said points, or ⊙s on the plan, with your scale, and see whether it agrees with the distance measured in the field or not. If it does, draw the line. Then lay your scale along it, and mark off the ⊙s, &c. as before.

Note.—The two sides or legs of any triangle are reduced to fast lines, and laid down as such, by first constructing the triangle by Prob. 13, section 1.

EXAMPLE.

Let it be required to lay down upon a plan, the line and dimensions entered in the following field book for a base line.

		2600	⊙ 6
		2560	+ ÷
		2000	⊙ 5
		1940	× ÷
		1600	⊙ 4
		1460	× ÷
		1200	⊙ 3
		1140	× ÷ left off.
	20	1080	
	40	950	
	90	810	to ∠
		700	× ÷
		610	⊙ 2
		400	× ÷
	60	280	left off.
	50	140	
⊙ 1	20	00	



METHOD OF OPERATION.

The line A B being drawn at pleasure, ⊙ 1st made, &c. (by the directions) at 610 links mark down ⊙ 2; at 1200 links ⊙ 3; at 1600 links ⊙ 4; at 2000 links ⊙ 5; and at 2600 links, or the end of the line, ⊙ 6. Next make a small dot or mark on the chain line at 140, 280, 400, 700 links, &c. Lastly, from ⊙ 1 lay off, to the left hand, an offset of 20 links: from the mark at 140, one of 50 links; and from that at 280, another of 60 links: also from that at 810, one of 90 links; from that at 950, one of 40 links; and from that at 1080, one of 20 links. Draw a line through the extremity of each offset (according to the sketching in the field book,) then is the line, &c. laid down upon the plan.

In the very same manner are the dimensions laid down, that are taken in measuring any other line, due regard being had to the field book, &c.

Note.—It is the chief practice, of most surveyors, to transfer to the plan, all the lines measured in a day, &c. before they begin with the offsets.

PROBLEM 5.

To lay down or plan an Angle taken with the Chain.

DIRECTIONS.

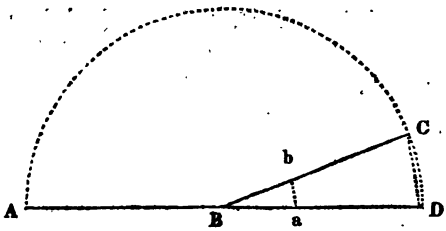
Make every link, in the length measured to obtain the required angle, 10 links; *i. e.* multiply the radius and chord by 10; then, by the note to prob. 13, section 1, with the tenfold dimensions make the required angle, and it is done.

Note.—When any of the lines that include the angle, are to extend a great length from the angular point, make every link as many as you can; *i. e.*

multiply the radius and chord, not only by 10, but by 20, 30, or 40, just as the plan will admit of; for the larger you make the sides that contain the angle, the more accurate will the angle be constructed on the plan.

EXAMPLE.

Let it be required to plan an angle taken with the chain, the chord of its supplement being 22.30 links, and radius 100.



Since B a and b a are given, we have, by the directions, D C $=$ 22.30 $\times$ 10 $=$ 223 links, and B D $=$ 100 $\times$ 10 $=$ 1000. Then, (by note, prob. 13, section 1, part 1,) from the center B, and radius B D, describe the semicircle D C A. Also from the center D, and radius D C, describe an arc to intersect the semicircle. Through the point of intersection C, draw the line B C; then will A B C be the angle required.

Note.—Although I have shewn both how to take an angle with the chain, and likewise how to plan it, I always avoid that practice, if I can only obtain proper dimensions another way; or if the lines, containing the angle, have to extend to any great distance from the angular point.

PROBLEM 6.

To reduce, with the Horn Ruler, a Field, whose sides are crooked, to one whose sides are straight lines, so that the Area inclosed by the latter, whether triangle, trapezium, or an irregular Figure, shall be nearly equal to that inclosed by the former.

DIRECTIONS.

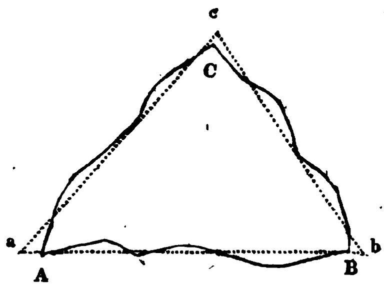
Apply your horn ruler to one of the crooked sides, in such a manner, that the area of the parts cut off the field,

by drawing a line along the edge of it, may, as near as you can judge, be equal to the area of the parts taken in, which are seen through the ruler. Apply it so to all the other sides, until you have done. Then will you have the crooked-sided field reduced to either a triangle, trapezium, or an irregular figure, of the same area nearly.

Cor.—Hence it is evident, that crooked fences may be reduced in the field, (by the eye) to straight ones, and surveyed as such.

EXAMPLE.

Let it be required to reduce, by the horn ruler, the crooked-sided field A B C, to a straight-lined one of the same area.

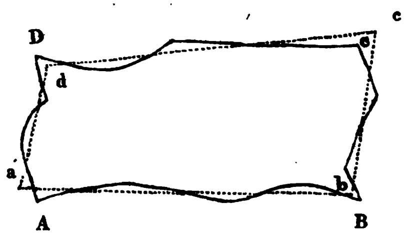


By applying the horn ruler as directed, we have the crooked-sided field A B C, reduced to the triangle a b c, or straight lined figure required.

EXAMPLE 2.

Let it be required to reduce, by the horn ruler, the

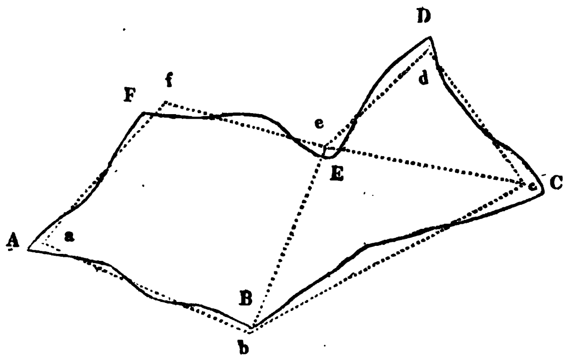
crooked sided field A B C D, to a straight-lined one of the same area.



Here, by applying the horn ruler to the crooked sides, as before, we have the field A B C D reduced to the trapezium a b c d, or right-lined figure required.

EXAMPLE 3.

Let it be required to reduce, by the horn ruler, the irregular field A B C D E F A, whose sides are crooked, to one whose sides are straight.



Here, by applying the horn ruler to the crooked sides of this field, as before, we have it reduced to the irregular one a b c d e f a, whose sides are straight lines. And by drawing e b, e c, it is reduced into a trapezium and two

triangles, viz. the trapezium a b e f, and the triangles b e c and d c e.

Note.—When any side is too long to be made straight at one operation with the horn ruler, make it straight at two, three, or more, as is most convenient.

Note 2.—This method of reducing crooked fences to straight ones, by the horn ruler, is used by all practical surveyors, and is as ready, and even as correct, as any that I know, in approximating the true area.

PROBLEM 7.

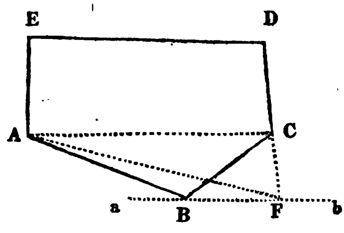
To reduce, by the Parallel Ruler, a Regular or Irregular Field, to a Triangle or Trapezium, of the same Area.

DIRECTIONS.

Take away the angles, (by prob. 23, section 1,) until you have reduced it to the figure wanted.

EXAMPLE.

Let it be required to reduce the irregular field A B C D E A, to a trapezium by the parallel ruler.



This is done the most readily by taking away the angle B. Therefore conceive the line A C drawn. Then, parallel to it, through angle B, draw the line a b to intersect the line D C, produced in F. Draw A F, then is the field A B C D E A reduced to the trapezium A F D E, of the same area.

Cor.—Hence, by taking away any one of the angles of the trapezium, the same figure or field will be reduced to a triangle of the same area.

Note.—In the very same manner is any other regular or irregular field or figure, reduced to a trapezium or triangle, be the number of sides what they may.

PROBLEM 8.

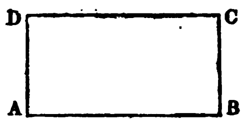
To find what Scale a Field is plotted from, viz. *whether* 1, 2, 3, 4, *&c. chains to an inch, from the Area and Plan of it.*

RULE.

Find the area of the field, by any scale whatever. Then, as that area is to the square of the scale calculated from, so is the given area to the square of the scale plotted from. —*(Emerson's Geo.* 8, 20.*)*

EXAMPLE.

Suppose the rectangular field A B C D, to contain four acres, or 400,000 square links, required the scale plotted from.



First, by a scale of three chains to an inch, its area is found $= 225000$ square links; whence $225000 : 9 :: 400000 : 16$ the square of the scale, consequently $\sqrt{16} = 4$ (chains to an inch,) the scale plotted from.

PROBLEM 9.

To reduce Statute Measure to customary, and vice versâ.

RULE I.

Multiply the number of perches in the area to be reduced, by the number of square feet in a square perch, statute measure; divide that product by the number of square feet in a square perch, customary measure, and it will give the area in perches, customary measure.

RULE II.

Multiply the number of perches in the area to be reduced, by the number of square feet in a square perch, customary measure; divide the product by the square feet in a square perch, statute measure, and it will give the area in perches, statute measure.

INVESTIGATION.

Let F and f be the number of feet in a perch statute and customary measure, and $p =$ the number of perches to be reduced; then (by prop. quoted in the last prob.) $F^2 : f^2 : :$ 1 perch statue $: \frac{f^2}{F^2}$ the quantity statute in a perch customary. Hence, by proportion, as 1 perch cus. $: \overset{\text{stat.}}{\frac{f^2}{F^2}} : : p.$ perches cus. $: \frac{p\,f^2}{F^2}$ statute, and by reversion $\frac{f^2}{F^2} : 1 : : p$ perches statute $: \frac{p\,F^2}{f^2}$ customary; hence the above rules.

Cor.—By taking the quotients of $\frac{F^2}{f^2}$ and $\frac{f^2}{F^2}$ for multipliers, and multiplying either the square perches, or square links to be reduced; you will likewise obtain the area in each case.

EXAMPLE 1.

Reduce 5A. 2R. 20P. statute, to customary measure, of 18 feet to a perch.

5A. 2R. 20P.	16.5	
4	16.5	
22	825	
40	990	
900	165	
	272.25	
	900	
		(4.0
18	324) 245025.00	(75.62
18	2268	
		4)18 36
144	1822	
18	1620	4 2 36
324	2025	
	1944	
	810	
	648	
	162	

Ans. 4A. 2R. 36P. customary.

EXAMPLE 2.

Reduce 4 A. 2 R. 36 P. customary, to statute measure.

16.5
16.5
825
990
165
272.25

4 A. 2 R. 36 P.
4
18
40
756
324
3024
1512
2268

18
18
144
18
324

272.25) 244944 (40
217800 (899
271440
245025
264150
245025
19125

4)22 19
5 2 19

Ans. 5A. 2R. 19P.

EXAMPLE 3.

Reduce 5A. 2R. 20P. statute, to customary, of 17.5 feet to a perch.

By the *Cor.* $f = 17.5$
17.5
875
1225
175
$f^2 = 306.25$

$F = 16.5$
16.5
825
990
165
$F^2 = 272.25$

306.25)272.25(.88898
245000

272500
245000

275000
245000

300000
275625

243750

5A. 2R. 20P.
4

22
40

900

.88898
900

4.0)80.08200
4)20.

5 0 0

Ans. 5 Acres.

EXAMPLE 4.

Reduce 5 Acres customary, of 17.5 feet to a perch, to statute measure.

$F^2 = 272.25)306.25 = f^2(1.125$
27225

34000
27225

67750
54450

13300

5A. 0R. 0P.
4

20
40

800

1.125
800

4.0)900.000

4)22.20

5 2 20

Ans. 5A. 2R. 20P.

Note.—By the preceding *Cor.* a table may soon be constructed whereby acres, roods, and perches statute, may be reduced to customary measure, (of any number of feet to a perch whatever,) and *vice versâ*; and it would be of as much use in a large survey of that nature, as the Land Valuer's Assistant is, for calculating the value of Estates. But I shall leave it to those whom it may concern, it being so variable and seldom of use.

PROBLEM 10.

To reduce Roods and Perches to the Decimal of an Acre.

RULE.

Divide the number of perches to be reduced by 160 (the perches in one acre,) and the quotient will be the decimal required.

Or, multiply .00625 (the decimal of one perch,) by the number of perches, and the product will be the decimal likewise.

EXAMPLE.

First 3 25
40

145

Then 145
.00625

725
290
870

Ans. .90625

Note.—Since no more than three places of decimals are ever used, (after the acres are dotted off to the left hand,) it is to be observed, when the fourth figure exceeds the number 5, that unity, or one, must be added to the third. When it does not exceed that number, take the three next to the acres as they are found above; hence, by adding and rejecting, we have the following table from one perch to 159, or 3R. 39P.

TABLE.

Perches.	Decimals	Roods and Perches.		Decimals	Roods and Perches.		Decimals	Roods and Perches.		Decimals
		R.	P.		R.	P.		R.	P.	
00	.000	1	0	.25	2	0	.500	3	0	.750
1	.006	1	1	.256	2	1	.506	3	1	.756
2	.013	1	2	.263	2	2	.513	3	2	.763
3	.019	1	3	.269	2	3	.519	3	3	.769
4	.025	1	4	.275	2	4	.525	3	4	.775
5	.031	1	5	.281	2	5	.531	3	5	.781
6	.038	1	6	.288	2	6	.538	3	6	.788
7	.044	1	7	.294	2	7	.544	3	7	.794
8	.050	1	8	.300	2	8	.550	3	8	.800
9	.056	1	9	.306	2	19	.556	3	9	.806
10	.063	1	10	.313	2	10	.563	3	10	.813
11	.069	1	11	.319	2	11	.569	3	11	.819
12	.075	1	12	.325	2	12	.575	3	12	.825
13	.081	1	13	.331	2	13	.581	3	13	.831
14	.088	1	14	.338	2	14	.588	3	14	.838
15	.094	1	15	.344	2	15	.594	3	15	.844
16	.100	1	16	.350	2	16	.600	3	16	.850
17	.106	1	17	.356	2	17	.606	3	17	.856
18	.113	1	18	.363	2	18	.613	3	18	.863
19	.119	1	19	.369	2	19	.619	3	19	.869
20	.125	1	20	.375	2	20	.625	3	20	.875
21	.131	1	21	.381	2	21	.631	3	21	.881
22	.138	1	22	.388	2	22	.638	3	22	.888
23	.144	1	23	.394	2	23	.644	3	23	.894
24	.150	1	24	.400	2	24	.650	3	24	.900
25	.156	1	25	.406	2	25	.656	3	25	.906
26	.163	1	26	.413	2	26	.663	3	26	.913
27	.169	1	27	.419	2	27	.669	3	27	.919
28	.175	1	28	.425	2	28	.675	3	28	.925
29	.181	1	29	.431	2	39	.681	3	29	.931
30	.188	1	30	.438	2	30	.688	3	30	.938
31	.194	1	31	.444	2	31	.694	3	31	.944
32	.200	1	32	.450	2	32	.700	3	32	.950
33	.206	1	33	.456	2	33	.706	3	33	.956
34	.213	1	34	.463	2	34	.713	3	34	.963
35	.219	1	35	.469	2	35	.719	3	35	.969
36	.225	1	36	.475	2	36	.725	3	36	.975
37	.231	1	37	.481	2	37	.731	3	37	.981
38	.238	1	38	.488	2	38	.738	3	38	.988
39	.244	1	39	.494	2	39	.744	3	39	.994

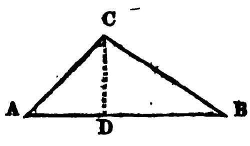
SECTION III.

OF SURVEYING AND CALCULATING THE AREA OF ANY SINGLE FIELD, WITHOUT PLOTTING.

A field is said to be surveyed, and its area calculated without plotting, when the area is found by the rules of mensuration, from the dimensions taken in the field: *i. e,* for a triangle, by measuring its base and perpendicular, or all its three sides; for a square, the length of one of its sides; for a parallelogram, an end and a side; for a trapezium, a diagonal, and two perpendiculars let fall therefrom to its opposite angles; and for a trapezoid, its two parallel sides, and the perpendicular distance between them. So find the area of each, by the respective rule it belongs to in Section 2, Part 1.

PROBLEM 1.

To survey and calculate the Area of the Triangular Field A B C.



METHOD OF SURVEYING.

Fix a mark in each corner of the field, if there be not a natural one, as in A, B, and C; then, from the corner A, measure along the side A B, until you come to the point D, where the perpendicular C D, falling from the

opposite angle C, intersects it; at which place leave a mark. Then complete the line A B, and note down its length in your book. Next return to the mark at D, and measure the perpendicular D C, also note down its length. Then is the field surveyed.

Note.—The point D is determined by prob. 3, section 2.

FIELD BOOK.	
Base A B = 830	Perp. D C = 300.

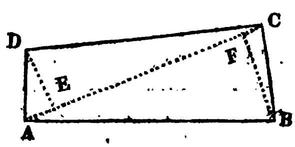
METHOD OF CALCULATION.

By prob. 2, section 2, part 1, $\frac{AB \times DC}{2} = \frac{830 \times 300}{2}$ $= \frac{249000}{2} = 124500$ square links = 1 acre, 0 roods, 39 perches, the area.

Note.—When the field is a regular polygon, of more than four sides, first find the center, as before directed, and in it fix a pole; then measure the length of one of its sides, and also the perpendicular let fall from the center upon the middle of the said side. The area then will be equal to the product of the perpendicular, multiplied by half the side measured, multiplied by the number of sides the figure contains.—(*Rule* 1, *prob.* 6, *section* 2, *part* 1.)

PROBLEM 2.

To survey and find the Area of the Trapezium A B C D, or any other Four-sided Field whatever.



METHOD OF SURVEYING.

First, fix a mark in every corner of the field, as before; then measure from the point A, along the diagonal A C, until you come to the point E, (where the perpendicular DE falls upon it from its opposite angle D,) at which place leave a mark. Proceed from E along the diagonal again, to the point F, (where the perpendicular B F falls upon it from its opposite angle B,) and fix a mark in it. Next continue the diagonal to the end or corner C, and note down its length. Then return to the points F and E, and measure therefrom the perpendiculars F B and E D, note down the length of each, and it is done.

FIELD BOOK.	
Diag. A C = 1070,	Per. F B = 380
	E D = 270

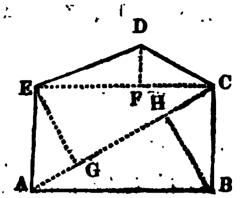
METHOD OF CALCULATION.

By prob. 4, section 2, part 1, $\frac{FB + ED}{2} \times AC = \frac{380 + 270}{2} \times 1070 = \frac{695500}{2} = 347750$ square links, equal 3 acres, 1 rood, and 36 perches, the area required.

Note.—As fields in the form of squares, parallelograms, &c. are very seldom met with in practice, I make it a general rule, to measure all four-sided fields by one and the same method, *viz.* the above, when the area is to be found without planning.

PROBLEM 3.

To survey and calculate the Area of the Irregular Field A B C D E A.



METHOD OF SURVEYING.

Fix a mark in every corner of the field, and suppose the line E C drawn. Then is the irregular field reduced to the triangle E C D, and the trapezium A B C E, which two parts, so divided, measure by the directions in prob. 1st and 2nd; then is the irregular field surveyed.

Note.—If the field to be surveyed, has ever so many sides, it is still to be divided into trapeziums and triangles, and surveyed as such, unless the number of sides are reduced by taking offsets to them, &c. as will be shewn hereafter.

FIELD BOOK.

Base E C = 770.	Perp. D F = 160
Diag[l]. A C = 900.	Perp[s]. { H B = 380, G E = 350 }

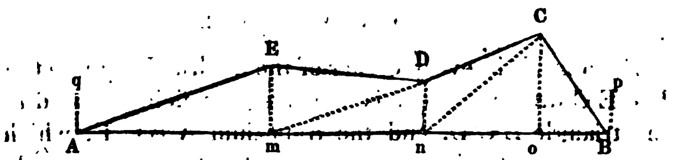
METHOD OF CALCULATION.

By prob. 2. section 2, part 1, $\frac{EC \times FD}{2} = \frac{770 \times 160}{2}$ $= \frac{123200}{2} = 61600$ square links, the area of the triangle

E C D; and, by prob. 4, ibid $\frac{HB + GE}{2} \times AC = \frac{730 \times 900}{2}$ $= \frac{657000}{2} = 328500$ square links, the area of the trapezium A B C E. Hence the sum of the two areas $= 328500 + 61600 = 390100$ square links, equal 3A. 3R. 24P. the area of the irregular field A B C D E A.

PROBLEM 4.

To survey and calculate the Area of the Long Irregular Field A B C D E A, by means of Offsets.



Take an offset from the point A, where you begin your chain line, if required. Then measure along the straight side A B, and take one to every angle in the crooked side, as from m, n, o, to E, D and C; note down the length of each, and its distance from the point A, where you take it, likewise the length of the line in the following manner; then is the field surveyed.

FIELD BOOK.

	00	1130	B
	210	980	150
	110	730	250
	140	400	330
A	00	00	400
			1130

METHOD OF CALCULATION.

From the field book, we have the following parts given, *viz.*

Links.	Links.
A q = 00	Also A m = 400
m E = 140	m n = 330
n D = 110	n o = 250
o C = 210	o B = 150
p B = 00	

Consequently, by prob. 3, section 2, part 1, we have $\frac{Aq + mE}{2} \times Am = \frac{00 + 140}{2} \times 400 = 28000$ links, the area of the part A m E. Also $\frac{mE + nD}{2} \times mn = \frac{140 + 110}{2} \times 330 = 41250$, the area of the part m n D E. Likewise $\frac{nD + oC}{2} + no = \frac{110 + 210}{2} \times 250 = 40000$, the area of the part n o C D; and $\frac{oC + pB}{2} \times oB = \frac{210 + 00}{2} \times 150 = 15750$, the area of the part o B C. Hence the sum of all the four areas $= 28000 + 41250 + 40000 + 15750 = 125000$ square links, equal 1 acre, 1 rood, the area of the whole irregular field A B C D E A.

Note 1.—The distances A m, m n, &c. are readily obtained, after the field is surveyed, by beginning at the top of the book, or end of the chain line, and subtracting every separate distance entered there, from the distance entered after it, and placing them opposite their respective places, as in the above field book; the sum of which parts, brought out by subtraction, is equal to the whole length of the line measured.

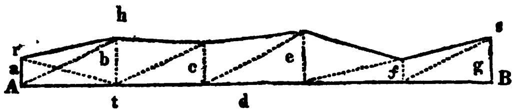
Note 2.—When the offsets are taken at equal distances, one from another, we have the following rule for calculating the area; which is far more commodious than that adopted above; and is the only one I ever remember

seeing, that would give a true answer. It was given me, about 30 years ago, by my worthy and learned friend Mr. John Rodham, of Richmond, Yorkshire, but without a demonstration. I take the liberty of inserting it here, with its demonstration likewise; to keep the learner from the old erroneous method of adding all the offsets together, and dividing by the number of them for a mean, when at equal distances, which is absolutely wrong.

RULE.

To half the sum of the first and last offset, add the sum of all the other offsets; divide the last sum by the number of offsets less one, for a mean; multiply the mean by the length of the chain line, (or sum of the distances between the offsets,) and the product will be the area required.

DEMONSTRATION.



Let a b c e f g represent six offsets, taken from the line A B at (d) distance one from another; A = the required area; and x equal the mean. Then, by the method of trapezoids,

$$\frac{a+b}{2}\times d+\frac{b+c}{2}\times d+\frac{c+e}{2}\times d+\frac{e+f}{2}\times d+\frac{f+g}{2}\times d=\frac{da+db+db+dc+dc+de+de+df+df+dg}{2}$$

$$=\frac{da+2db+2dc+2de+2df+dg}{2}=A=5dx,$$

from whence, by transposition and division, is had $\frac{a+g}{2}+b+c+e+f=5x$, consequently $\frac{\frac{a+g}{2}+b+c+e+f}{5}=x$, the mean, and $5dx=A$, the area.* Q. E. D.

* For a similar demonstration, see Hutton's Course, vol. 2, page 41.

Cor.—The area is likewise equal $\left(\frac{a+g}{2}+b+c+e+f\right)\times d$; for since $\frac{da+2db+2dc+2de+2df+dg}{2}=A$, it is evident, after dividing by 2, it is reduced to $\left(\frac{a+g}{2}+b+c+e+f\right)\times d=A$.

Cor. 2.—When the first and last offsets are each nothing, that is when the line A B is the chord of the crooked fence A r s B, (see the preceding fig.) the area is then truly expressed by $\frac{b+c+e+f}{5}\times 5d$ (or A B); which expression is the same as that given by Professor Vince, in his Trigonometry, Art. 146, page 78; from whence we have the following Rule.

RULE 2.

Add all the offsets together, and divide them by their number increased by unity; multiply the quotient by the length of the chord or line, measured with the chain, and it will give the area.

EXAMPLE.

Required the area of the irregular piece of ground A B s r A, from the following field book.

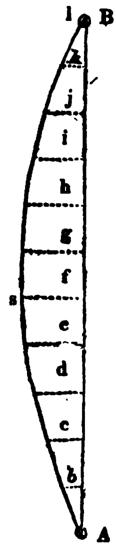
	Links.
a =	40
b =	110
c =	90
e =	120
f =	70
g =	100
d =	200

By Rule 1, $\dfrac{\frac{a+g}{2}+b+c+e+f}{5} = \dfrac{\frac{40+100}{2}+110+90+120\,c+70}{5} = 92$, the mean, and $5\,d \times 92 = 1000 \times 92 =$ 92000 square links, equal 3 roods and 27 perches, the area.

EXAMPLE 2.

Required the area of the irregular piece of ground A B s A, from the following field book.

		⊙ B
	l = 00	1100
	k = 40	1000
	j = 90	900
	i = 115	800
	h = 130	700
	g = 140	600
	f = 140	500
	e = 130	400
	d = 100	300
	c = 80	200
	b = 40	100
⊙ A	a = 00	00
	11) 1005	
	91.363	



By Rule 2, $\dfrac{b+c+d+e+f+g+h+i+j+k}{11}$

$$= \frac{40+80+100+130+140+140+130+115+90+40}{11}$$

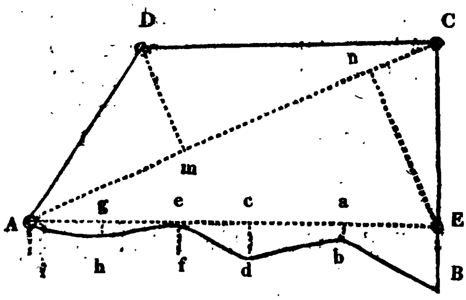
$= \frac{1005}{11} = 91.363$ the mean; and $91.363 \times 1100 = 100499$ square links, equal to 1 acre, the area.

Note.—By reckoning the first and last offset, each one, to obtain the divisor, when they are nothing (as above,) the first rule will give the true answer, in both cases.

Note.—The area of narrow pieces of ground, that taper regularly, either to a point or not, is found, by taking half the sum of the breadths at each end, and multiplying it by the length, through the middle.

PROBLEM 5.

To survey and calculate the Area of the Irregular Field A B C D, whose side A B is very crooked.



METHOD OF SURVEYING.

Fix a mark in every corner of the field; also one in the side B C, where you can see the mark at A from it, as at E. Then measure from the mark at E, to that at A, and take the offsets E B, a b, &c. as in prob. 4; then is the part A B E surveyed. Next measure the trapezium A E C D, as directed in prob. 2, and its done.

FIELD BOOK.

Diag. A G = 940. Perps. $\begin{cases} n\,E = 350 \\ m\,D = 230 \end{cases}$		00	880	A
		40	710	
		00	550	
		70	400	
		30	200	
	E	140	00	

METHOD OF CALCULATION.

From the field book, we have given the following dimensions; *viz.*

E B = 140	Also Æ a = 200
a b = 30	a c = 200
c d = 70	c e = 150
e f = 00	e g = 160
g h = 40	g A = 170
A i = 00	

Likewise the diag^{l}. A C = 940, and the Perps. $\begin{cases} n\,e = 350 \\ m\,d = 230 \end{cases}$

Hence, by proceeding as in prob. 4,

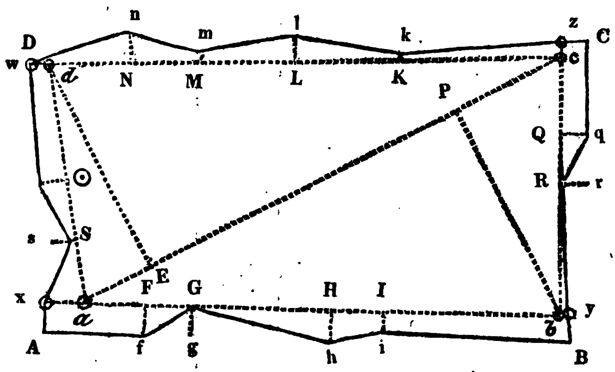
$$\frac{\overline{Eb+ab}\times Ea+\overline{ab+cd}\times ac+\overline{cd+ef}\times ce+\overline{ef+gh}\times eg+\overline{gh+Ai}\times gA}{2}=\frac{\overline{140+30}\times 200+\overline{30+70}\times 200+\overline{70+0}\times 150+\overline{0+40}\times 160+\overline{40+0}\times 170}{2}$$

$$= \frac{34000 + 20000 + 10500 + 6400 + 6800}{2} = \frac{77700}{2}$$

$= 38850$, the area of the part A B E. Also $\frac{n\,E + m\,D}{2}$ $\times$ C A $= \frac{350 + 230}{2} \times 940 = \frac{545200}{2} = 272600$, the area of the trapezium A E C D; whence the sum of the two areas, $38850 + 272600 = 311450$ square links, equal to 3 acres and 18 perches, the area of the field A B C D.

PROBLEM 6.

To survey and calculate the Area of the Field A B C D, whose sides are all crooked.



METHOD OF SURVEYING.

Fix a mark in or near every corner of the field, as at a b c and d, so that you can see from one to another, along each side of the field, as from that at a to that at b, from that at b to that at c, &c. Make ⊙ x upon the fence A D, in a line with the marks at a and b, and measure from thence to ⊙ y; from b through c to ⊙ z; from c through d to ⊙ w; and from d to a. Take the offsets F f, G g, &c. as before:

then are all the parts between the inscribed trapezium and the boundary of the field surveyed. Next measure the trapezium a b c d, by prob. 2; then is the whole field A B C D surveyed, which was required.

FIELD BOOK.

	515	80	a
	385	0	
	280	55	
d	00	43	
	1123	20	⊙ w
	1080		d
	910	72	
	778	15	
	580	70	
	350	00	
c	00	30	
	570	60	⊙ z
	540		c
	380	70	
	290	00	
b	00	30	
	1130	64	⊙ y
	1100		b
	730	40	
	620	75	
	300	00	
	220	75	
	80		a
⊙ x	00	60	

Diag. a c = 1150. Perp. $\begin{cases} \text{F b} = 483 \\ \text{E d} = 480 \end{cases}$

METHOD OF CALCULATION.

From the field book are given the following parts, viz.

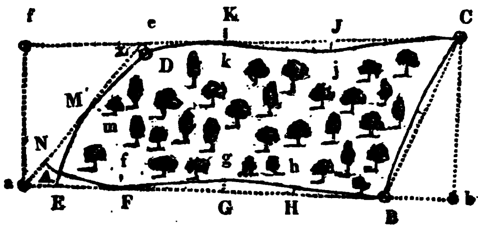
	x F = 220	x A = 60
	F G = 80	F f = 75
	G H = 320	G g = 00
	H I = 110	H h = 75
	I y = 400	I i = 40
	b R = 290	y B = 64
	R Q = 90	b y = 30
	Q z = 190	R r = 00
	c K = 350	Q q = 70
	K L = 230	z C = 60
	L M = 198	c z = 30
	M N = 132	K k = 00
	N W = 213	L l = 70
	d O = 280	M m = 15
	o S = 105	N n = 72
	S a = 130	w D = 20
		d w = 43
Diagl. a c = 1150. Perp.	P b = 483	O o = 55
	E d = 480	S s = 00
		a x = 80

Hence, by proceeding as in the last Problem, we have the area of the part A B y x = 56975; the area of the part y C z b = 19850; the area of the part z c w D = 37255; and the area of the part w d a x = 21807. Also, the area of the trapezium a b c d = 553725, from whence the sum of the five areas = 553725 + 21807 + 37255 + 19850 + 56975 = 689612 square links, = 6 acres, 3 roods, and 23 perches, the area required.

Note.—When you circumscribe a field, or even part of it, with either a triangle or a trapezium, the several areas between the boundary of the field, and chain lines, must then be deducted from the area of the circumscribing figure, in order to obtain the true area of the said field.

PROBLEM 7.

To survey and calculate the Area of the Wood A B C D.



METHOD OF SURVEYING.

Having fixed a mark at or near every corner of the wood, begin at any place where you can see along two sides, to the marks fixed in the corners, as at a, and measure therefrom along the side A B to B, at which place leave a mark; then produce the said line, until a perpendicular will fall from C upon the part produced, as at b; then measure from b to C, and from C to B; next go back to C, and measure therefrom, along the side C D to e. Produce the line C e to f, where a perpendicular, from a, will fall. From f, measure to a, and from a to e, and it is done; the offsets being taken all along, as before.

Note.—By circumscribing the wood with the two right angled triangles a b C and C f a, having one hypothenuse common to both, it not only determines the truth of the dimensions, but also the sides of the wood; the two lines a e, B C, likewise determine the ends, and prove that the two points or ⊙s e and B, are truly entered. For a clearer demonstration see further on.

FIELD BOOK.

		400	00	e
		380	10	
		190	00	
		60	4	
a		00	00	
f		380		a
		925		f
	20	670		e
	00	510		
	25	310		
C	00	00		
		385		B
		200	20	
C		00	00	
b		350		C
		910		b
	00	754		B
	30	505		
	33	380		
	00	113		
	30	60		
a	00	00		

Base a b = 910 Perp. b C = 350.
Base C f = 925 Perp. f a = 308.
Base B b = 156 and Base f e = 255.

METHOD OF CALCULATION.

From the field book we have the following dimensions, *viz.*

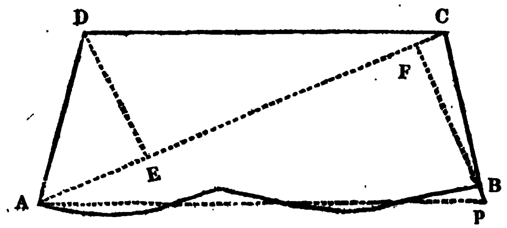
a E = 60	E A = 30
E F = 53	F f = 00
F G = 267	G g = 33
G H = 125	H h = 30
H B = 249	B = 00
B I = 185	I i = 20
I C = 200	C = 00
C J = 310	J j = 25
J K = 200	K k = 00
K e = 160	e d = 20
e L = 20	L D = 4
L M = 190	M m = 00
M N = 130	N A = 10
N a = 60	

The base a b = 910, and the perpendicular b C = 350 of the right angled triangle a b C; also the base C f = 925, and perpendicular f a = 308, of the right angled triangle a f C; likewise B b, f e = 156 and 255, the perpendiculars of the two right angled triangles B b C, and a f e. Hence, then, by Prob. 2, sec. 2, p. 1, $\frac{ab \times bC}{2} = \frac{910 \times 350}{2} = 1.59250$, the area of the triangle a b C; and $\frac{Cf \times fa}{2} = \frac{925 \times 308}{2} = 1.42450$, the area of the triangle a b C; their sum $1.59250 + 1.42450 = 3.01700$, the area of the circumscribing fig. a b C f. Also, $\frac{Bb \times bC}{2} + \frac{ef \times fa}{2} = \frac{156 \times 350}{2} + \frac{255 \times 308}{2} = .66570$, the area of the two triangles B b C, and e f a; and, by

proceeding as in Prob. 6, we have the area of the part a A B a = 13772; the area of the part B C i B = .03850; the area of the part C D e C = .07975; and the area of the part e a A D = .01370. The area of all the four parts, with the area of the two right angled triangles B b C and e f a = 66570 + 13772 + 03850 + 07975 + 01370 = .93537, equal the area between the circumscribing figure a b C f and the wood. Hence the area of a b C f, minus the area of all the parts circumscribed, = 3.01700 — .93537 = 2.08163 square links, equal to 2 acres and 13 perches, the area of the wood.

PROBLEM 8.

To survey and find the Area of the Irregular Field A B C D, without offsets.



METHOD OF SURVEYING.

Fix a mark in every corner of the field. Then produce the line or side C B, until a line drawn from its extremity, as from p to A, will give and take equal quantities; at p fix a mark; then is the field A B C D reduced to prob. 2, *viz.* to the trapezium A p C D, and surveyed as such.

FIELD BOOK.

Diag[l.] A C = 930	Perp[s.] { D E = 300 F p = 360

METHOD OF CALCULATION.

From the field book, we have, in the trapezium A p C D, the diagonal A C = 930, the perpendiculars D E = 300, and F p = 360 links. Hence, $\frac{DE + Fp}{2} \times AC = \frac{300 + 360}{2} \times 930 = \frac{613800}{2} = 3.06900$ square links, equal 3A. 0R. 11P. the area of the field A B C D.—*(Prob. 4, sec. 2, pt. 2.)*

Note.—Although the above method is not so true as those given in the preceding Problems, neither can it be so often applied in practice, on account of large fences, &c. that obstruct the eye from seeing every separate crank when looking along them; yet, a practical surveyor, by it, will approximate the area of a piece of ground so nearly, as to differ very little from the truth. Consequently, by a little practice, any other person that is but little acquainted with surveying, will soon be able to determine the point p, as above; or any other point of the like nature. For, by looking from one pole to another, as from the pole at p to that at A, it is very easy to judge, in a short distance, when the parts taken in (by supposing a line drawn between the two poles) are equal, or nearly so, to the parts left out. Hence, when all the fences of a field are to be made straight, they are done in the same manner, one after another. But when any one fence, to be made straight, is either so long, or so crooked, that it cannot be all seen when standing at one end, make it straight at two, three, or more operations; then measure the field, by the Problems it is reduced to, and it is done.

Note.—I have, as yet, only touched upon the taking of proper dimensions, and finding the area therefrom, by the rules of mensuration, without paying any regard as to proving the truth of the said dimensions. Now, although the above is the only method used by practical surveyors, and is the best, provided no error be committed in taking the dimensions; yet, since none are so infallible but they may make mistakes, I think it is necessary that some other line or lines should be introduced, to answer the same purpose as proof lines do in the plan of a field. I shall therefore conclude this section, by laying down a method of taking dimensions to the above Problems, and proving them to be right, before we begin our calculations, or even leave the field.

The new Method of taking Dimensions and proving them, in order to obtain the area of any field, without planning it.

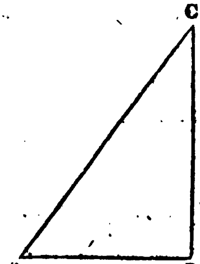


ILLUSTRATION.

Suppose A C D to represent a right angled triangular field, to be surveyed, and its area required, without planning; it is very well known, that in order to perform that operation, we must have the true lengths of A D and C D, or the perimeter of the figure or field; as either sort of dimensions will give the true area, if taken right, agreeably to what is laid down in all books of mensuration. Consequently, suppose I have measured the lengths of A D and D C; to prove that the said lengths or dimensions are taken right, I will measure the side A C likewise; and, before I begin to calculate the area, will try, (by Euclid 47.1,) whether $\sqrt{AD^2 + DC^2}$ be equal to the length of A C, measured on the ground or not. If it is, I say my dimensions are taken right, and by the above rules I can bring out the true area. But if $\sqrt{AD^2 + DC^2}$ is not equal to A C, I say some of my dimensions are wrong; and to find out the error as soon as possible, I measure the hypothenuse A C over again, and find it to be right, and that the error is either in the base or perpendicular; which lines I likewise measure over until I find out the mistake. Now, as they are the only two lines from which we chiefly make our calculations, it is evident, by trying the line A C with

Euclid 47.1, it not only saves me from committing an error, but causes me to be able, with proper calculations, to bring out a true and correct area—an area that will stand the scrutiny of the greatest surveyor whatever. Hence, the necessity of measuring such lines as above, to prove the data, is manifest.—For the truth of the above illustration, see *Euclid Data Prop.*

New method of Measuring Problem 1, or the Plain Triangle A B C.

Fix a mark in every corner of the field, as in the old method, if there is not a natural one; then begin at the corner A, and measure towards B, until you come to the point D, where the perpendicular C D has to be erected, at which place leave a mark, and note down in your book its exact distance from A. Afterwards continue your line, until you come to B, and note down D B likewise. Return to D, and measure the perpendicular D C; after noting down its length, measure the two sides A C and B C as proof lines; note down their lengths, and you have done.

FIELD BOOK.

A D = 270
B D = 560
D C = 300
A C = 403
B C = 635

METHOD OF PROVING.

First, to prove the truth of the lines A D, B D, and C D, we must have, by Euclid 47.1, $\sqrt{AD^2+DC^2} =$ A C, or its equal $\sqrt{270^2+300^2} = 403$, and $\sqrt{BD^2+DC^2} =$ C B, or its equal $\sqrt{560^2+300^2} = 635$. Hence,

THE OPERATION.

270	300	560
270	300	560
18900	DC^2 90000	33600
540		2800
$AD^2 =$ 72900		$DB^2 =$ 313600
$DC^2 =$ 90000		90000
162900(403 = AC proof.		)403600(635 = BC proof.
16		36
803) 2900		123) 436
2409		369
491		1265) 6700
		6325
		375

Note.—As no parts of links are ever taken into account, and the above lengths of A C and B C, found by Euclid, are so near their lengths measured upon the ground, I take it for granted, that the data, or lengths of A D, B D, and DC, are measured right, and consequently, then begin to make my calculations.

Cor.—When the field is a square, after measuring a side, by measuring the diagonal, it not only proves the side measured, but the whole of them, by reason of their being equal to one another.

Cor. 2.—When the field is a parallelogram, after measuring an end and a side, by measuring the diagonal, it not only proves the end and side measured, but it proves the other end and side likewise; because both ends, as well as the sides, are equal to each other.

Cor. 3.—When the field is a regular polygon, after measuring a side, and the perpendicular from the middle of the side to the centre, by measuring a line from the

centre to either extremity of the side, it proves the whole triangle, because the perpendicular is erected on the middle of the side.

New Method of Surveying Problem 2, or the Trapezium A B C D.

After fixing a mark in every corner of the field, begin and measure the diagonal A C, and note down in your book the respective lengths of A E, A F, F C, and C E; also leave a mark at E and F. After you have completed this, return to F and E, and measure the perpendiculars B F, E D, and likewise note down their lengths. Lastly, for proof lines, measure the three sides A B, B C, and C D, note down their lengths; then is the field surveyed.

FIELD BOOK.

A E	=	270
A F	=	840
F C	=	230
C E	=	800
B F	=	380
E D	=	270
A B	=	922
B C	=	444
C D	=	844

METHOD OF PROVING.

First, to prove that the diagonal and perpendiculars are taken right, we must again have, by Euclid 47.1, the $\sqrt{AF^2 + FB^2} = AB$, or its equal $\sqrt{840^2 + 380^2} = 922$; the $\sqrt{FB^2 + FC^2} = BC$, or its equal $\sqrt{230^2 + 380^2} = 444$; and the $\sqrt{CE^2 + ED^2} = DC$, or its equal $\sqrt{800^2 + 270^2} = 844$. Hence the

OPERATION.

840
840
33600
6720
$AF^2 = 705600$
144400
850000(922 = A B proof.
81 · ·
182) 400
364
1842) 3600
3684

380
380
30400
1140
$FB^2 = 144400$

230
230
6900
460
$FC^2 = 52900$
144400
197300(444 = B C proof.
16 · ·
84) 373
336
884) 3700
3536
164

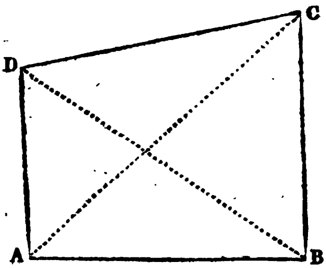
800
800
$CE^2 = 640000$

270
270
18900
540
$ED^2 = 72900$
640000
712900(844 = C D proof.
64 · ·
164) 729
656
1684) 7300
6736
564

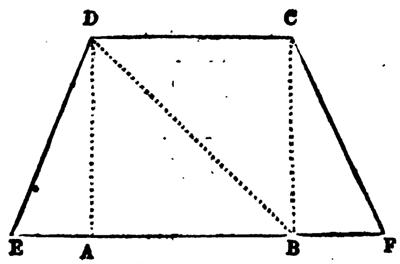
Cor.—By measuring the three lines A B, B C, and C D, is proved the whole of the data; for, since the two lines A B and B C proved both the diagonal and the perpen-

dicular F C, it, of course, must follow, that D C proved the position and magnitude of D E, which were all the data required.

Note.—When any one root does not vary more than a single link, from the line measured in the field, you may take it for granted that your data or dimensions are right.



Cor.—When the field or figure is a trapezoid, whose parallel sides stand perpendicular to its base, as the trapezoid A B C D, by measuring the two diagonals A C and B D, after we have measured A D, A B, and B C, they will prove the whole data; for, since D B proves both D A and A B, it follows that A C proves both C B and A B likewise.



Cor. 3.—When the trapezoid is constructed upon its longest parallel side, as the trapezoid E F C D, by measuring the lines F C, D B, and D E, after the respective

lengths of E A, A B, B F, B C, and A C are measured and noted down, may be proved the whole data; for, since F C and E D prove both the base and the perpendicular of their respective triangles, it follows that B D proves both the shorter ‖ side D C, and the part A B, by reason of the parallels.

New Method of Surveying Problem 3, or the Irregular Field A B C D E A.

A mark being fixed in every corner, and the field supposed to be divided into a trapezium and triangle, by the line E C, first measure and note down in your field book, the respective distances on the diagonal and base, where the perpendiculars are erected; also the lengths of the perpendiculars, as in the two preceding problems; then, to prove that the dimensions are taken right, measure the sides A B, B C, and C D; then is the field surveyed.

FIELD BOOK.

A G = 215
A H = 600
H C = 300
C G = 685
H B = 380
G E = 350
E F = 430
F C = 340
F D = 160
A B = 710
B C = 484
C D = 375
E C = 770

METHOD OF PROVING.

Now to prove that the diagonal, base, and perpendiculars of this figure are taken right, we must again have, (by Euclid 47.1,) the $\sqrt{AH^2 + HB^2} = AB$, or its equal

$\sqrt{600^2 + 380^2} = 710$; the $\sqrt{GH^2 + HB^2} = BC$, or its equal $\sqrt{300^2 + 380^2} = 484$; the $\sqrt{CG^2 + EG^2} = EC$, or its equal $\sqrt{685^2 + 350^2} = 770$; and the $\sqrt{CF^2 + FD^2} = DC$, or its equal $\sqrt{340^2 + 160^2} = 375$.

OPERATION.

```
             600      380                 300
             600      380                 300
         -------    -----               -----
AH² = 360000      30400        CH² = 90000
     --------     1140              144400
                 -------            -------
        HB² = 144400               234400(484 = BC
              360000               16··
             -------               -----
             504400(710 = AB    88)744
             49··                  704
             ----                  ----
         141) 144              964)4000
              141                  3856
             ----                  ----
         142) 300                   144
             ----                  ----

        685           350           340         160
        685           350           340         160
       ----          ----          ----        ----
       3425         17500         13600        9600
      5480          1050          1020         160
     4110          ------        ------       ------
     ------  EG² = 122500  CF² = 115600  FD² = 25600
CG² = 469225        ------          25600       ------
      122500                       ------
     -------                       141200(375 = DC
     591725(769.2 = EC nearly.     9··
     49··                          ----
     ----                       67)512
 146)1017                          469
      876                          ----
     ----                      745)4300
1529)14125                         3725
     13761                         ----
     -----                          575
15382) 36400                       ----
       -----
```

New Method of Surveying Problem 4, or the Long Irregular Field A B C D E A, by means of Offsets.

Measure along the straight side A B, take the offsets, note down their lengths, and also the distances A n, m n, &c., that one offset is from another, as directed in the old method; but be careful to leave a mark at A, m, n, &c., as well as at C, D, and E, when you take the offsets. When you have measured to B, to prove that your dimensions are taken right, measure from B to C, from C to n, from D to m, and from E to A, note down their respective lengths, and it is done.

FIELD BOOK.

A	m	=	400
m	n	=	330
n	o	=	250
o	B	=	150
m	E	=	140
n	D	=	110
o	C	=	210
B	C	=	258
C	n	=	326
D	m	=	348
E	A	=	423

METHOD OF PROVING.

Now, to know the truth of the dimensions, or data, we must have, as before, by Euclid 47.1, the $\sqrt{\text{A m}^2 + \text{m E}^2} = \text{A E}$, or its equal $\sqrt{400^2 + 140^2} = 423$; the $\sqrt{\text{m n}^2 + \text{n D}^2} = \text{m D}$, or its equal $\sqrt{330^2 + 110^2} = 348$; the $\sqrt{\text{n o}^2 + \text{o C}^2} = \text{n C}$, or its equal $\sqrt{250^2 + 210^2} = 326$; and the $\sqrt{\text{o C}^2 + \text{o B}^2} = \text{C B}$, or its equal $\sqrt{210^2 + 150^2} = 258$. Hence, the

OPERATION.

```
        400  140            330
        400  140            330     110
        ---  ---            ---     110
A m²=160000 5600           9900     ---
             140            990    1100
            ----           ----     110
   m E²=19600    m n²=108900       ----
          160000          n D²=12100
          ------                 108900
          179600(423=A E proof   ------
          16                     121000(348=m D proof.
          --                     9
      82)196                     --
          164                 64)310
          ---                    256
     843)3200                    ---
         2529                688)5400
         ----                    5504
         .671

      250          210                150
      250          210                150
      ---          ---                ---
    12500         2100               7500
    500            420                150
    ----          ----               ----
n o²=62500  o C²=44100          o B² 22500
                 62500                44100
                 -----                -----
                106600(326=nC proof. 66600(258=CB proof
                 9                    4
                 --                   --
              62) 166             45) 266
                  124                 225
                  ---                 ---
              646)4200           508)4100
                  3876               4064
                  ----               ----
                   324                 36
```

Note.—In proving such Figures as belong to this Problem, it must be understood, that if the first and last piece, instead of being triangles had been trapezoids, as in Fig. 2, to this Problem, it would have been necessary to measure both the diagonals r t, A H, in the first trapezoid; after that, only one in each of the other, as is there drawn.

Problems 5 and 6, fall under the methods laid down to problems 2 and 4, and the above note.

Problem 7. See the figure and method laid down above.

Problem 8, falls under the method laid down to problem 2.

Note.—Since any field, whose sides are straight lines, can be divided into plain triangles and trapeziums, be the number of sides what they may; it follows, that such field may be divided into right angled triangles likewise, and the data proved as above.

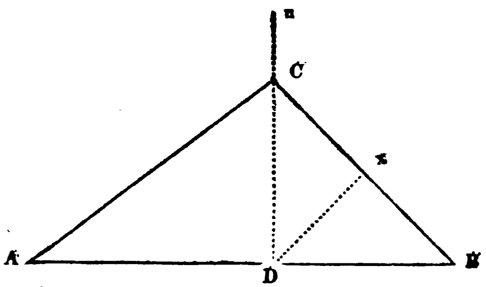
SECTION IV.

OF SURVEYING AND PLANNING ANY SINGLE FIELD WHATEVER.

By surveying and planning a single field, is meant, the taking of proper dimensions upon the ground, and laying them down upon paper, by the Problems of Geometry, so as to form an exact figure of the said field.

PROBLEM 1.

Let A B C represent a Triangular Field to be surveyed and planned.



METHOD OF SURVEYING.

First fix a pole or flag in every corner of the field; then begin at one of them, as at A, and measure along the side A B, but be careful to observe where the perpendicular C D will fall, and enter the distance A D in your book;

leave a mark at D, until you have completed the whole line A B, which length likewise enter into your book; next measure the perpendicular D C, after that A C and B C, note down their lengths, and it is done.

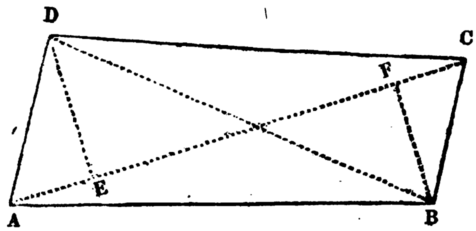
CONSTRUCTION OR METHOD OF PLANNING.

On any line, drawn at pleasure, lay down, from your field book, the distance between A and B; also mark off the point D in its proper place. From D, erect the perpendicular D n; on D n mark off D C, its exact length; draw C B, C A, and it is done; or, you may construct it, by prob. 13, sec. 1, part 1.

Note.—By measuring all the three sides, the plan and area of the field might have been likewise obtained; but to prove that the survey was correct, it would have been necessary to measure the perpendicular likewise, or some other line between two fixed points, as the line D x. Hence, the reason of my measuring, and entering into the field book, the lines A C and B C, above.

PROBLEM 2.

Let A B C D represent a Trapezium or any other four-sided Field, to be surveyed and planned.



METHOD OF SURVEYING.

Having fixed a mark in every corner of the field, first measure the diagonal A C, and mark the places E and F where the perpendiculars fall; also note down in your

book their respective distances from A. Next measure the sides A B, D C, and the two perpendiculars E D and F B, also the two ends A D, B C, or the diagonal B D, note down their lengths, and it is done,

METHOD OF PLANNING.

The method of planning this, or any other four-sided field, is the same as that laid down in the last problem; the diagonal being a base common to both triangles.

PROBLEM 3.

To measure and plan any regular Polygon.

METHOD OF SURVEYING.

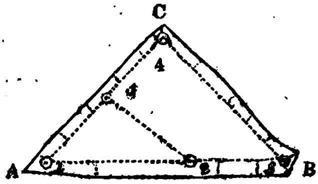
By problem 19, sec. 1, part 1, first find the center of the polygon, and fix a mark therein; then measure the length of one of its sides, and the perpendicular or distance from the center to the middle of the side; likewise the radius of its circumscribing circle; note down in your book, their respective lengths, and it is done.

METHOD OF PLANNING.

Since the base, or side of the polygon, the perpendicular erected on the middle of the side, and the radius of its circumscribing circle are given, it falls under the construction to prob. 17, sec. 1, part 1.

Note.—The method of keeping the field book, for the above three problems, being so extremely simple and easy, I have omitted them; as any entry of the proper lengths and distances, that the surveyor can understand himself, will answer the purpose required.

Let A B C represent an irregular Piece of Ground to be surveyed and planned.



FIELD BOOK.

⊙ 2		230		⊙ 5
		390	14	⊙ 1
		340	24	
		250	16	
		210		⊙ 5
		130	17	
		50	18	
⊙ 4 S.		00	16	
		380	15	⊙ 4
		300	14	
		200	16	
		100	10	
		60	15	
⊙ 3 N.		00	45	to 7 or corner.
		530	18	⊙ 3
		440	26	
		360	20	
		300		⊙ 2
		100	25	
⊙ 1 E.		00	20	

METHOD OF SURVEYING.

In any corner, as in A, make ⊙ 1 ; then look to the other corners B and C, and see if there is any object in the direction of which you can measure, so as to keep you in a straight

line: if there is not, a flag must be placed near them, as in ⊙ 3 and ⊙ 4; also leave another at ⊙ 1. Then measure separately the distance between every two of them; also the offsets from the chain line to every crank in the fence, as directed in prob. 1, sec. 2, part 2: but in measuring the distance between ⊙ 1 and ⊙ 3, as well as the distance between ⊙ 4 and ⊙ 1, observe to make ⊙ 2 and ⊙ 5, and measure the distance between them likewise, which distance note down in your book, and it is done, if the lines are measured right.

CONSTRUCTION OR METHOD OF PLANNING FROM THE FIELD BOOK.

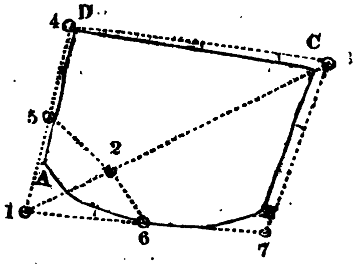
On any line drawn at pleasure, lay down, from your book, the distance between ⊙ 1 and ⊙ 3, (by prob. 4, sec. 2, part 2.) Then with ⊙ 1 as a center, and radius the distance between ⊙ 1 and ⊙ 4, describe a fine arc. Also, with ⊙ 3 as a center, and radius the distance between ⊙ 3 and ⊙ 4, describe another arc to intersect the former in ⊙ 4. From ⊙ 4, draw lines to ⊙ 1 and ⊙ 3. Then will you have laid down the triangle measured in the field. Next, make ⊙ 2 and ⊙ 5, in their proper places, and draw the line between them. Lay your scale on each of the chain lines, mark off the offsets in their proper places, and erect their perpendicular lengths thereon; afterwards, through their extremities, draw the fences, and it is done.

Note.—If the distance between ⊙ 2 and ⊙ 5, upon the plan, be not the same as that measured in the field, the dimensions are taken wrong; consequently the lines must then be measured over again.

Note.—If there had been no offsets, the figure would have been a triangle, as well as the following one a trapezium; consequently they are reduced from such to irregular figures, by means of offsets.

PROBLEM 5.

Let ABCD represent a Field to be surveyed and planned.



FIELD BOOK.

⊙ 6		120		⊙ 2
		400		⊙ 3
	20	230		
⊙ 7	12	60		Pasd. ⌝
		500		⊙ 7
	00	350		Left off
		250		⊙ 6
⊙ 1 S. E.	20	170		
⊙ 2		180		⊙ 5
		410		⊙ 1
	00	300		Left off.
		200		⊙ 5
⊙ 4. S.	10	100		
		590		⊙ 4
	10	580		Pasd. ⌝ or corner.
⊙ 3 W.	15	208		
		740		⊙ 3
		698		+ ÷ at ⌝
		200		⊙ 2
⊙ 1 E.		80		+ ÷ at ⌝

METHOD OF SURVEYING.

Having got a visible object in or near every corner of the field, as in the preceding problem, measure from ⊙ 1 to ⊙ 3, for a base line; but observe, in measuring it, to make ⊙ 2, and enter it, as well as every hedge you cross, &c. into your book. Then measure from ⊙ 3 to ⊙ 4, and from ⊙ 4 to ⊙ 1; make ⊙ 5 somewhere between the two latter, and measure the distance between it and ⊙ 2; then have you completed one side of your base line. Next measure from ⊙ 1 to ⊙ 7, and from ⊙ 7 to ⊙ 3; also the proof line between ⊙ 2 and ⊙ 6, and it is done;—the offsets being taken all along from the chain line, to every crank in the fence, &c. as before directed.

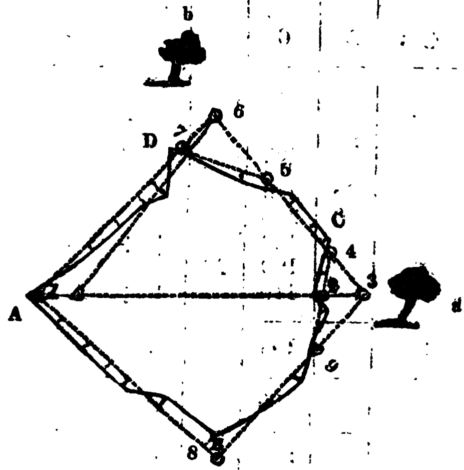
METHOD OF PLANNING.

On any line drawn at pleasure, lay down, from your book, the length of the base line, *i. e.* the distance between ⊙ 1 and ⊙ 3; also make ⊙ 2 in its right place: then is the field reduced to two triangles, having one base (*viz.* the distance between ⊙ 1 and ⊙ 3,) common to them both. All the other sides being given likewise, it is planned exactly in the same manner as the preceding problem.

Note.—Although the line or distance between ⊙ 2 and ⊙ 5, and between ⊙ 2, and ⊙ 6, is of no use in planning or finding the area of this, or the preceding problems, yet it is necessary to have such line or lines in every single field, to prove that the dimensions of the inscribed or circumscribing triangles, are taken right.—(See note to preceding problem.) But in large surveys, they are always of use, helping both to find the area of each field, and to prove that the dimensions are taken right; owing to their extending frequently along the sides of fences, as well as from one side of a triangle or trapezium, to its opposite side; and the most preferable are those that intersect the other lines, at right angles, as will be shewn hereafter.

PROBLEM 6.

Let A B C D A represent an irregular Field to be surveyed and planned.



FIELD BOOK.

		130		⊙ 2
		120		+ ÷
		100	20	
⊙ 9		60		+ ÷
		474		⊙ 3
		305		⊙ 9
		270		+ ÷ left off.
		254	15	
⊙ 8. N.		213		+ ÷
		524		⊙ 8
	32	503		to ∠ or corner.
	40	362		
		315		+ ÷
		305	10	
		245	18	
		200	24	
		140	16	
⊙ 1 S.		00	15	

		225		⊙ 7
⊙ 5	16	85		
		100		⊙ 4
⊙ 2	20	60		
		530	15	⊙ 1
		500		+ ÷
	30	400		
	25	310		
	25	230		
	50	200		
		130		+ ÷
		120	20	
		110		+ ÷
⊙ 6 S.		100		⊙ 7
		500		⊙ 6
		300		⊙ 5
		270		+ ÷
		250	25	
		210	10	
		160	10	
		150		+ ÷
⊙ 3. N.		130		⊙ 4
		700		⊙ 3
		600		⊙ 2
⊙ 1 E.		580		+ ÷ at ∠

METHOD OF SURVEYING.

In any corner of the field, as at A, make ⊙ 1. Then measure from it in the direction of some visible object, as of the tree a, for a base line; continuing your line always until you think a line drawn from it, to somewhere near the corners D and B, will do the most work in the simplest manner, as from ⊙ 3 to ⊙ 6, in the direction of the tree b.

The base line being thus measured, &c., next measure from ⊙ 3 to ⊙ 6, and make ⊙ 4 and ⊙ 5, where most convenient. Then measure from ⊙ 6 to ⊙ 1, and make ⊙ 7. Lastly, measure from ⊙ 2 to ⊙ 4, and from ⊙ 5 to ⊙ 7, then is one side of the base line completed. Next measure from ⊙ 1 towards the corner B, in the direction of some particular object as before, to ⊙ 8; and from ⊙ 8 to ⊙ 3; also make ⊙ 9 between them; then measure from ⊙ 9 to ⊙ 2, and it is done; the offsets, &c. being taken as before.

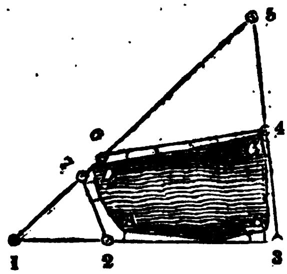
Note.—It is to be understood, that in the above method of proceeding, a flag is left in ⊙ 1, and another in ⊙ 3, from which to measure in the direction of ⊙ 6 and ⊙ 8—a plan very useful in short distances. But in some cases, where the distances are great, it is rendered useless, unless the line is ranged with poles. To supply this defect, I make it a rule to measure in the direction of any visible object, that will carry me near the place wanted, so as to cross some fixed line, (or line before measured); as from ⊙ 6 to the trigon in the line, between ⊙ 1 and ⊙ 2. Then from the trigon I measure to the nearest ⊙ in the said line, as ⊙ 1, and note down its distance, East, West, North, or South. Then is the line determined, as to position, the same as if measured to a ⊙; and it is always to be understood, through this book, when a line is measured from a ⊙, and not terminated in another, that it is measured in this manner.

METHOD OF PLANNING.

The method of planning this field from the field book, is exactly the same as that given to the preceding problem.

PROBLEM 7.

Let A B C D represent a Mere or Pond of Water to be surveyed and planned.



FIELD BOOK.

		350		⊙ 4
		240	16	
		180	20	
		100	10	
⊙ 6		50	15	
		145		⊙ 7
		130	15	
⊙ 2		100	20	
		700		⊙ 1
		500		⊙ 7
	00	460		Pasd. ÷
⊙ 5 S.		440		⊙ 6
		480		⊙ 5
		240		⊙ 4
	22	220		Pasd. ÷
⊙ 3 N.	22	100		
		560		⊙ 3
	15	540		Pasd. ÷
	20	500		
	00	460		
	20	230		Pasd. ÷
⊙ 1 E.		200		⊙ 2

METHOD OF SURVEYING.

Make ⊙ 1 in such a place, that a line drawn from it to ⊙ 5, passing by the point D, will be the shortest and most convenient possible. Leave a flag or mark, and measure from it to ⊙ 3; then measure from ⊙ 3 in the direction of some visible object, along the side B C, until you can see the flag in ⊙ 1, as to ⊙ 5. Next measure from ⊙ 5 to ⊙ 1; afterwards measure the proof lines, and it is done, the offsets being taken, &c.

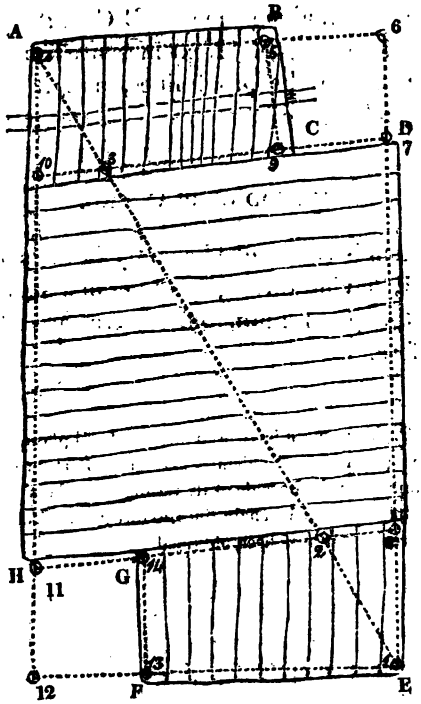
METHOD OF PLANNING.

This problem is constructed exactly in the same manner as problem 1, having three given lines or distances to make a triangle; the proof lines, offsets, &c. are likewise laid down as before.

Note.—This Problem might have been measured and planned, by proceeding in the same manner as in Problem 7, Section 3, Part 2; or—by measuring proper distances on the outside of it, taking the offsets and angles, (or their supplements) made by the chain lines, as directed in Section 1, Part 2—the figure and area might have been obtained.

PROBLEM 8.

Let A B C D E F G H A represent an open Township Field, to be surveyed and planned.



FIELD BOOK.

⊙ 3		150 120 70 20		⊙ 10 Left and ent. Left and ent. Left and ent.
⊙ 14		255 125	20	⊙ 13

	610		⊙ 11
	510	16	
	390	14	+ ÷
	380		⊙ 14
	340		Left and ent.
	290	15	
	240		
	190		Left and ent.
	140	22	
	90		
⊙ 2	40		Left and ent.
	765	20	⊙ 1
	730		
	680		
	630		
	580		
	530		Left and ent.
	480	20	
	430		
	380		
	330		
	280		left & ent. field land
	250		⊙ 13
⊙ 12 E.	235	20	+ ÷ ent. balk.
	1330		⊙ 12
	1080		⊙ 11
	1065		left headland & + ÷
	1000		left and ent.
	950		
	900	28	
	850		
	800		
	750		
	700		Left and ent.
	650		
	600	30	
	550		
	500		
	450		
	400		
	350		Left and ent.
	300		Ent. head land.
	260	15	⊙ 10
⊙ 4 S.	00	8	

		233		⊙ 5
⊙ 9		95		+ Rd. & left tongue
		600		⊙ 3
		565		
		515		
	30	465		
		445		
		420		
		395		
		370		
		320		
	24	275		
		225		Left and ent.
		210		⊙ 9
	15	185		+ ÷
⊙ 7	20	80		
		150		⊙ 2
		140		Left and ent.
		90		Left and ent.
⊙ 8		40		Left and ent.
	10	1320		⊙ 1
	16	1180		
		1020		⊙ 8
	15	990		Left head land.
		940		
		890		
		840		
	21	790		
		740		
		690		
		640		Left and ent.
	20	590		
		540		
		490		
		440		
	22	390		
		340		
		290		Left and ent.
	15	235		+ ÷
⊙ 6 S.		210		⊙ 7

		750		⊙ 6
	30	550	10	+ ÷ left tongue.
		525		⊙ 5
		500		
		450		
		400		
		375		
		350		
		325		Left and ent.
	20	300		
		250		
		200		
		150		
		100		
		50		Left and ent.
⊙ 4 E.	20	00		
		1510		⊙ 4
		1425		Left and ent.
		1340		Leftand ent. + Rd.
		1245		Left and ent.
		1215		⊙ 3
		1184		left head land & ent.
		1120		
		1070		
		1018		
		960		
		900		
		845		
		790		Left and ent.
		740		
		685		
		630		
		570		
		515		
		455		
		400		Left and ent.
		345		Left furw. and ent. head land.
		310		⊙ 2
		280		Left and ent.
		170		Left and ent.
⊙ 1 N.		85		Left and ent. fur.

METHOD OF SURVEYING.

Having fixed a mark in or near every corner of the field, choose the most convenient direction for your base line, as from ⊙ 1 near the corner E, to ⊙ 4 near that of A. Then measure the line; but particularly observe to take and enter into your field book, the respective distance between each furrow, or partition between one ridge and another, from the beginning of your line or ⊙ 1; also make ⊙s. where most convenient. Take offsets and make remarks where necessary, until you complete your line. Having arrived at ⊙ 4, measure from thence Eastwards to ⊙ 6; and, near the corner B, make ⊙ 5. From ⊙ 6, measure Southwards to ⊙ 1, and make the intermediate ⊙s. *viz.* 7 and 8. From ⊙ 8 measure to ⊙ 2, and from ⊙ 7 measure to ⊙ 3, and make ⊙ 9 near the corner C. From ⊙ 9 measure to ⊙ 5, then have you completed the East side of your base line.

Next, from ⊙ 4 measure South to ⊙ 12; from ⊙ 12 East to ⊙ 1; and make the intermediate ⊙s. *viz.* 10, 11, and 13. From ⊙ 2 measure to ⊙ 11, and make ⊙ 14. From ⊙ 14 measure to ⊙ 13; and lastly from ⊙ 3 measure to ⊙ 10, and it is done;—every furrow or partition, between ridge and ridge, in every line measured, as well as the offsets and other remarks, being all along duly noticed and entered in the field book.

Note.—The boundary of every head land or ridge, which other ridges abut upon, must be traced the same as a boundary fence of an old inclosure.

METHOD OF PLANNING.

The method of planning the above field is extremely easy, on account of the lands being both short and straight, due regard being had to the field book, and what has been done before. For, after the crossings of the furrows, or partitions

between ridge and ridge, are marked, with a small dot, upon every line that intersects them, there is nothing more to do, than to draw a fine line through the said marks, in every flat or range of lands; as by so doing, the furrows or partitions between ridge and ridge will be truly laid down. The ends are also determined by lines drawn through the extremities of the offsets erected on the chain lines.

Note.—The above field or figure is the simplest or easiest I could invent, for the learner to begin with. Where the ridges are very crooked and long, or connected with a number of common balks or pieces of land which taper to a point, called a tongue, extending only part of the length of the flat, it is necessary to measure lines in different directions, and take offsets to the partition, furrows, &c. the same as when tracing out the fence of an old inclosure, in order to obtain the position and true figure of every piece or parcel of land —an operation which is very tedious.

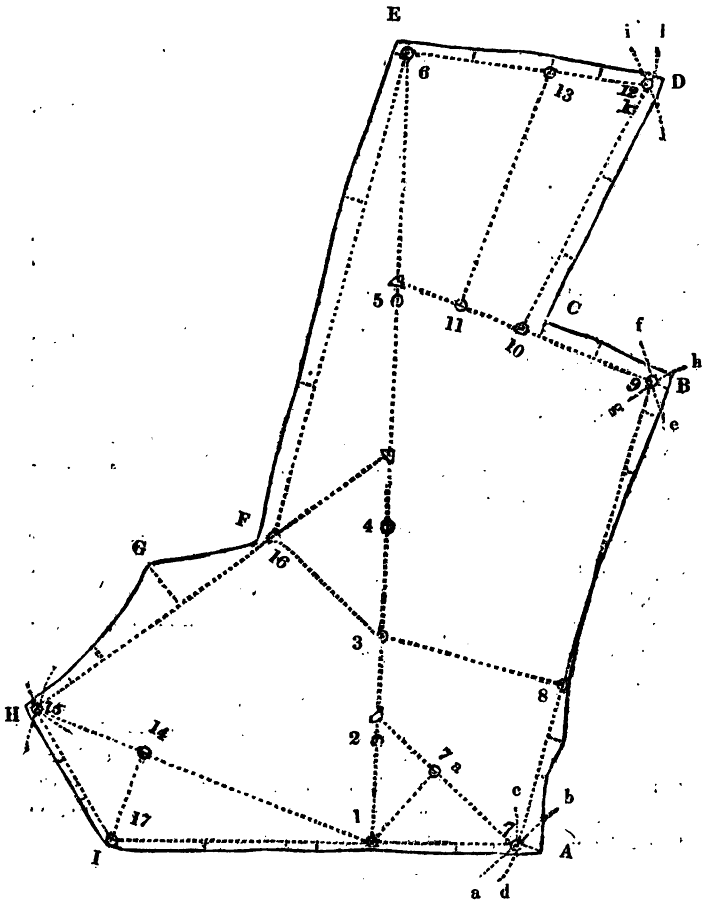
If it had been required to measure the above field for an inclosure, I should have measured and planned it in flats only, and, from the area of each, have found the quantity each proprietor had in the said flat, as will be shewn further on. This method will answer for any inclosure, where the ridges bear a certain proportional breadth one to another, *viz.* when two narrow ones are equal to one broad ditto, as in flat 1 above, and when the flats are likewise clear of balks, &c.

By the word flat, mentioned in this problem, I mean a certain number of ridges that lay between one noted place and another; *i. e.* between a foot path, known by a certain name, and a balk or ridge known by another certain name, as flat 2 above, extending between the ends of the ridges called Butts, and those called House Lands.

Note.—Having only touched in this Section, upon Problems that can be circumscribed or inscribed with one triangle or trapezium—although it frequently occurs in practice, that two, three, four, or more, are obliged to be constituted on each side of the base line, to procure proper dimensions for planning a field when large, or a township—I shall, by way of example, introduce a problem of that description.

PROBLEM 9.

Let I A B C D E F G H I be an Irregular Field or Common, to be surveyed and planned.



FIELD BOOK.

	20	1054		⊙ 6
	48	710		
	45	310		
⊙ 16	38	00		
⊙ 17		190		⊙ 14
	15	310		⊙ 15
	10	180		
	10	100		
⊙ 17 N.	20	00		
	12	550		⊙ 17
	13	350		
	10	200		
⊙ 1 W.	10	124		
⊙ 16		310		⊙ 3
		920		△ 145 L N. of ⊙ 4
		620		⊙ 16
	10	575		Pasd. ∠
	110	370		
	30	190		
	25	100		
⊙ 15 E.	24	00		
		755		⊙ 15
⊙ 1st. W		510		
⊙ 11		525		⊙ 13
		510	24	⊙ 6
		398	26	
		250	28	⊙ 13
		110	25	
⊙ 12 W.		00	23	

		585		⊙ 12
		570	30	
		380	24	
⊙ 10 N.		175	25	
⊙ 3		395		⊙ 8
		580		△ 30 L. N. of ⊙ 5.
		430		⊙ 11
		295		⊙ 10
		260	24	Pas. ∠
		120	29	
⊙ 9 W.		00	25	
		1010	34	⊙ 9
		770	10	
		525	00	
		385	00	
		345		⊙ 8
		240	33	
		130	10	
⊙ 7 N.		00	30	
⊙ 7 a		200		⊙ 1
		400		△ 45 L. N. of ⊙ 2.
⊙ 7 W.		235		⊙ 7 a
		300	15	⊙ 7
		160	20	
⊙ 1 E.		00	30	
		1650		⊙ 6
		1130		⊙ 5
		655		⊙ 4
		420		⊙ 3
⊙ 1st. N. for base line.		210		⊙ 2

Note.—Although the sketching part in every field book, throughout this Section, is omitted, it is to be observed, that it is always practised when measuring in the field. The reason why I omitted it is, that the figures are only single fields, and contain both the chain lines and offsets, which will fully compensate for the omission, if the student duly compares the marks and characters entered into each book, with the explanations, &c. in Section 1, Part 2.

METHOD OF SURVEYING.

After measuring the base line as before directed, begin at ⊙ 1, and measure towards the outside of the estate, as Eastwards to ⊙ 7, near the corner A. Then measure from ⊙ 7 Westwards, until you cross the base line, as at the △ North of ⊙ 2. Next measure the proof line betweeen ⊙ 1 and ⊙ 7 a. Then you have completed the triangle 1, 7, △ N. ⊙ 2, and consequently got a fixed point at the outside of the estate or field.

Next, from ⊙ 7, the fixed point, measure along the fence A B Northwards, as far as is thought convenient, as to ⊙ 9. Then, from ⊙ 9, measure Westward again, until you cross the base line, at the △ North of ⊙ 5. Next measure the proof line between ⊙ 3 and ⊙ 8; then have you completed the trapezium 7, 9, △ N. 5, △ N. 2. Lastly, from ⊙ 10, measure again Northwards to ⊙ 12, and from ⊙ 12 to ⊙ 6, at the end of the base line. Next measure the proof line, between ⊙ 11 and ⊙ 13, then have you completed the trapezium 10, 12, 6, △ N. 5, and consequently all the work on the East side of the base line.

Then return to ⊙ 1, and measure Westwards to the corner H, near which make ⊙ 15. Then measure from ⊙ 15 Eastwards, until you cross the base line, as at the △ North of ⊙ 4; measure the proof line between ⊙ 16 and ⊙ 3; then have you completed the triangle 1, 15, △ N. 4. Next measure from ⊙ 1 to ⊙ 17, near the corner I, and from ⊙ 17 to ⊙ 15; also the proof line between ⊙ 14 and ⊙ 17; then have you completed the triangle 1, 17, and 15. Lastly,

measure from ⊙ 16 to 6, then have you completed the West side of the base line, and consequently got proper dimensions for plotting the field or common.

Note.—It is to be understood, that the offsets are taken to every crank in the fence, in this problem, the same as in the preceding problems, from the lines that run along the sides of the fences, &c.

Note 2.—In case of measuring a common for an inclosure, it is to be observed, that all old roads, mounds, and other remarkable objects upon the common, as well as all the gates, hedge-ends, &c. adjoining, must be traced and laid upon the rough plan.

Note 3.—Although the preceding fields are measured, &c. as irregular-lined ones, when they strictly are bounded by irregular curves; yet this method may be so applied, as to differ very little from the truth, by supposing, as above, the short distances between the extremities of every two offsets, a right line.

REMARK.

It is evident that, by varying the base line, the position and magnitude of the other lines will vary likewise; yet, by applying the method here laid down, you will still obtain the same plan and area. It follows, of consequence, that the plan of a field, or township, &c. may be obtained, an infinite number of ways, by one and the same method of proceeding. But, it is to be understood, that the base line must always be so chosen by the surveyor, as to have all the conveniences possible. The other lines likewise must be chosen or contrived to have the same conveniences also, by making the intermediate ⊙s. in proper places, when measuring; which ⊙s., in large surveys, are generally made near the crossing of fences, or opposite hedge ends, &c. for obtaining the true figure of each field, contained in the survey; by measuring from one ⊙ (or △ near to it) to another, and taking the offsets, &c. between them, until the whole survey is completed—that is, until every hedge, road, brook, house, &c. contained therein, are regularly traced.—*(See Plan.)*

METHOD OF PLANNING.

First lay down the base line from your book, as directed in Problem 4, Section 2. Then, with the center ⊙ 1, and radius the distance between ⊙ 1 and ⊙ 7, describe the arc d c. Next, with the radius or distance between ⊙ 7, and the △ N. of ⊙ 2, describe the arc a b, to intersect the former. Draw lines from the intersection of these two

arcs, to ⊙ 1, and the △ North of ⊙ 2, and you will have the three lines or sides of the triangle 1, 7, △ N. 2, laid down the same length as measured in the field: the intersection of a b and d c is ⊙ 7. Next make the intermediate ⊙, *viz.* ⊙ 7 a; draw the line between ⊙ 1 and it; then will be laid down all the chain lines in the first triangle, constituted on the East side of the base line.

Again, with the radius or distance between ⊙ 9 and ⊙ 7, and between ⊙ 9, and the △ North of ⊙ 5, and the centers ⊙ 7, and △ North of ⊙ 5, describe the two arcs g h, e f, to intersect each other, as in ⊙ 9. From the point of intersection, draw lines to the two centers; afterwards make the intermediate ⊙s. *viz.* ⊙ 8, ⊙ 10, and ⊙ 11; and draw the proof line between ⊙ 3 and ⊙ 8; then will be laid down all the chain lines in the trapezium △ N. ⊙ 2, 7, 9, △ N. ⊙ 5, or second figure constituted on the East side of the base line. Lastly, with the center ⊙ 10, and radius the distance between ⊙ 10 and ⊙ 12, describe the arc i j; also, with the center ⊙ 6, and radius the distance between ⊙ 12 and ⊙ 6, describe the arc k l, to cross the arc i j. Draw lines from the point of intersection, *viz.* ⊙ 12, to the two centers, and make ⊙ 13. Afterwards draw the proof line from ⊙ 11 to ⊙ 13, then will be laid down all the chain lines in the last trapezium or figure, constituted on the East side of the base line.

And in the very same manner, are all the triangles on the West side of the base line constructed, due regard being had to the dimensions taken and entered into the field book.

Having thus laid down, on the plan, all the chain lines measured in the field, return to the first line measured and entered in your book, and lay down all the offsets, &c. taken from it; afterwards to the second, third, &c.; proceeding always, line after line, just as entered in your book, according to the directions laid down in Problem 3,

Section 2, Part 2, until you have completed the plan of the survey.

Cor.—Hence, by using methods similar to those given in this Section, any other field or common may be surveyed and planned, though different, both in area and figure, *viz.* whether regular, irregular, or curved.

SECTION V.

OF FINDING THE AREA OF ANY FIELD, FROM THE PLAN OF IT.

Having, in the preceding Section, shewn how to survey and plan any field or figure, whether right-lined, regular, irregular, or curved, I shall now proceed to point out the method of finding their areas from the plan, by the Problems of Mensuration, after they are reduced, when necessary, to regular figures, by the horn and parallel rulers.

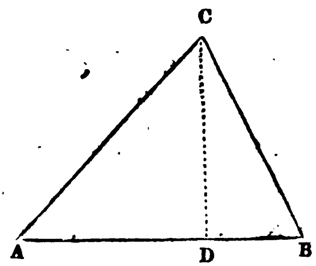
Case 1.—*When the field is a Triangle.*

RULE.

Let fall a perpendicular from the vertical angle upon the base; then measure both it and the base separately, by the scale from which the field was planned; and find its area by Problem 1, Section 2, Part 1.

EXAMPLE.

Required the area of the triangular field A B C, planned from a scale of 4 chains to an inch.



After letting fall the perpendicular C D, as directed by the Note to Problem 9, Section 1, Part 1, we find its length, by the scale planned from, = 430 links; also the base A B = 635; consequently, by the Problem above, $\frac{AB \times CD}{2} = \frac{635 \times 430}{2} = \frac{273050}{2} = 136525$ square links, equal 1 acre, 1 rood, and 18 perches, the area required.

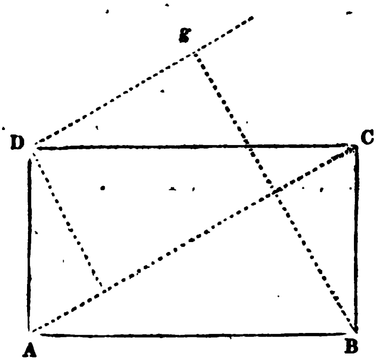
CASE 2.—*When the field is a square, rectangle, or trapezium.*

RULE.

First draw the diagonal; then let fall the perpendiculars upon it, and measure their lengths; after that, find its area by Problem 4, Section 2, Part 1.

EXAMPLE.

Required the area of the rectangular field A B C D, plotted from a scale of 4 chains to an inch.



By first drawing the diagonal, &c. as above directed, we find, by the scale planned from, B g = 750 links; also the diagonal A C = 980; hence, $\frac{B\,g}{2} \times A\,C = \frac{750}{2} \times 980 = 375 \times 980 = 367500$ square links, equal 3 acres, 2 roods, and 28 perches, the area required.

Note.—The readiest and most accurate method of finding the length of the two perpendiculars, is this:—Draw D g || to A C, then let fall B g perpendicular to A C or D g, and take its length at one operation, which will be equal to the sum of the two perpendiculars.

CASE 3.—*When the field is a regular one,* i. e. *a polygon, whose sides are all equal to one another.*

RULE.

Draw a line from the center of the figure to the middle of one of its sides; then measure its length with the scale planned from, also the length of a side, and find its area by Rule 1, Problem 6, Section 2, Part 1.

RULE 2.

Measure the length of a side, by the scale planned from, and find its area by Rule 2, Problem 6, ibid.

RULE III.

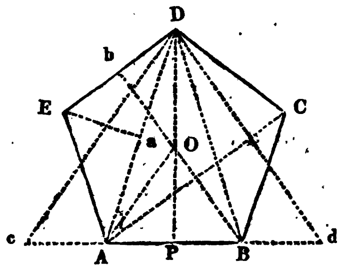
Divide the field into triangles and trapeziums by diagonal lines; then measure the parts it is divided into, as before directed, and find the areas by their respective cases.

RULE IV.

Take away the angles, by Problem 23, Section 1, Part 1, so as to reduce it to a triangle, or a trapezium; then measure it as such, by case 1 or 2.

EXAMPLE.

Required the area of the pentagonal field A B C D E A, planned from a scale of 4 chains to an inch, by Rule 1.



After drawing O P, we find its length 413 links, also the length of the side A B = 600; hence $\frac{O P \times A B}{2} \times 5$ $= \frac{413 \times 600}{2} \times 5 = \frac{1239000}{2} = 619500$ square links, equal 6 acres and 31 perches, the area required.

EXAMPLE.

Required the area of the above field, by Rule 2.

Since A B = 600 links, we have $A B^2 \times 1.7204774 = 600 \times 600 \times 1.7204774 = 619371$ square links, = 6 acres and 31 perches, nearly as before.

EXAMPLE.

Required the area, by Rule 3.

By drawing the diagonals A C and A D, the question falls under Cases 1 and 2. Hence, by letting fall the perpendiculars, we find, by the scale planned from, the base A D = diagonal A C = 975 links, E a = 350, also B b, the sum of the two perpendiculars let fall on the diagonal, = 925. Then, by Case 1, $\frac{A D \times E a}{2} = \frac{975 \times 350}{2} =$ 170625 square links, the area of the triangle A E D; and, by Case 2, $\frac{B b \times A C}{2} = \frac{925 \times 975}{2} = 450937$, the area of the trapezium A B C D; consequently their sum = $170625 + 450937 = 621562$ square links, equal 6 acres and 34 perches the area.

EXAMPLE.

Required the area, by Rule 4.

By taking away the angles, the field is reduced to the triangle D c d, falling under Case 1; from whence we find,

by the scale planned from, the base c d = 1344 links; also the perpendicular P D = 925; therefore $\frac{c\,d \times P\,D}{2}$ $= \frac{1344 \times 925}{2} = \frac{1243200}{2} = 621600$ square links, equal 6 acres and 34 perches the area.

Note.—The small variation in the two last examples, arises chiefly from taking the dimensions from the plan with a scale of small divisions; for the finest line of which we can make use, will be the breadth of two links or more, by a scale of four chains to an inch. Hence, a dispute may very soon arise, between two persons, respecting the area of a field taken from a plan, laid down from a scale of four, or even three chains to an inch.

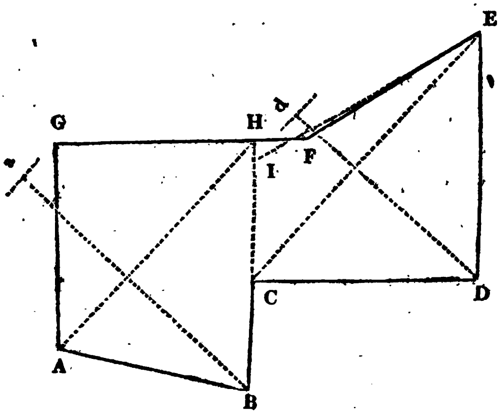
CASE 4.—*When the field is an irregular one.*

RULE.

Divide it into parts more commodious to be measured, by right lines drawn from side to side, or from angle to side, &c. Then take away any of the angles, so as to reduce the said parts to triangles or trapeziums, or both, and measure them as such by their respective Cases.

EXAMPLE.

Required the area of the irregular right-lined field A B C D E F G A, planned from a scale of 4 chains to an inch.



First, by producing the line B C to H, and taking away the angle F, we have the field reduced to the two trapeziums A B H G and C D E I, falling under Case 2. Hence, by the scale planned from, are had A H = 620, B a = sum of perpendiculars = 650, C E = 700, and D d = sum of perpendiculars = 510 links; consequently $\frac{Ba \times AH}{2} + \frac{Dd \times CE}{2} = \frac{650 \times 620}{2} + \frac{510 \times 700}{2} = \frac{403000}{2} + \frac{357000}{2} = 201500 + 178500 = 380000$ square links, equal 3 acres 3 roods and 8 perches, the area required.

It is to be observed that the short lines *a* and *d* are parallel to A H and C E, and would, if continued, pass through G and I.

CASE 5.—*When the field is a very irregular one,* i. e. *one whose sides are very small and numerous, which is frequently the figure found upon a plan, having its boundary or fence straight, between the extremities of every two offsets.*

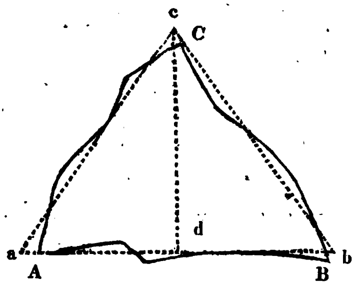
RULE.

Make the fences more straight with your horn ruler, by Problem 5, Section 2, Part 2, and afterwards by your parallel ruler, if required, until you have reduced the figure to some of the preceding Cases, by which find its area.

Note.—Although the above method or rule, (which is the only one used by surveyors,) cannot be looked upon as mathematically true; yet a practical surveyor, by it, will approximate the area of any field whatever, very near the truth, as will appear by duly observing the following figures.

EXAMPLE.

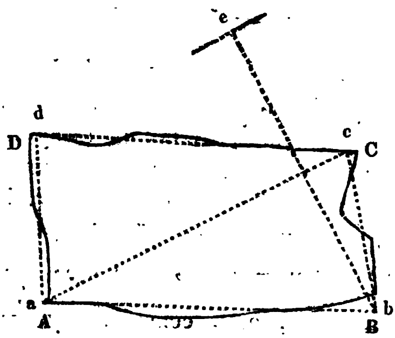
Required the area of the irregular or crooked-sided field A B C, planned from a scale of 4 chains to an inch.



First, by applying the horn ruler, the irregular field A B C is reduced to the triangle a b c, falling under Case 1; consequently, by the scale planned from, we have a b = 680, d c = 470; whence $\frac{a\,b \times d\,c}{2} = \frac{680 \times 470}{2} = \frac{319600}{2} = 159800$ square links, equal 1 acre 2 roods and 15 perches, the area required.

EXAMPLE 2.

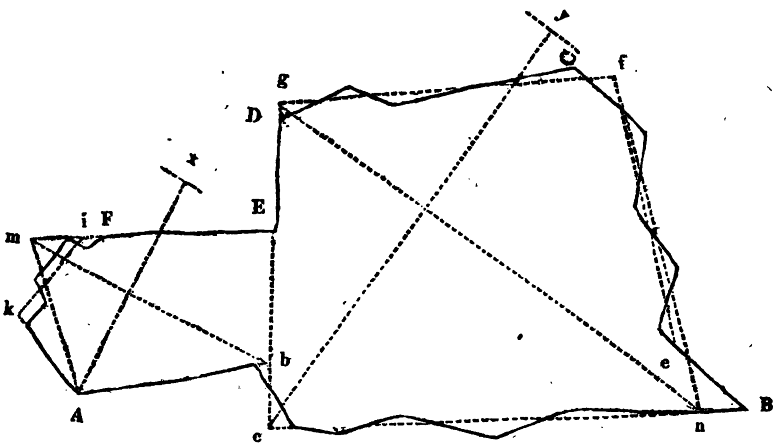
Required the area of the irregular or crooked-fenced field A B C D, planned from a scale of 4 chains to an inch.



First, by applying the horn ruler, the field A B C D, is reduced to the trapezium a b c d, falling under Case 2. Hence, by the scale planned from, we have a c = 720, b e, the sum of the two perpendiculars, = 660; consequently $\frac{a c \times b e}{2} = \frac{720 \times 660}{2} = \frac{475200}{2} = 237600$ square links, equal 2 acres 1 rood and 20 perches, the area required.

EXAMPLE 3.

Required the area of the irregular and crooked-sided field B C D E F A B, planned from a scale of 4 chains to an inch.

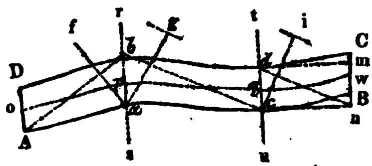


First, by applying the horn-ruler to the boundary of the given field, we have it reduced to the irregular one A b c B e f g E i k A. Next, by taking away the angles k and e, we have it reduced to the two trapeziums A m E b and c n f g, falling under Case 2. Hence, by the scale planned from, we have m b = 575; A x, sum of perpendiculars, = 520; g n = 1120; and c y, sum of perpendiculars, = 1005; and, consequently, $\frac{A x \times m b}{2} +$

$\frac{c\,y \times g\,n}{2} = \frac{520 \times 575}{2} + \frac{1120 \times 1005}{2} = 149500 +$ 562800 = 712300 square links, equal 7 acres and 19 perches, the area required.

EXAMPLE 4.

Required the area of the narrow strip of land A B C D, planned from a scale of 4 chains to an inch.



METHOD.

By drawing the lines r s, t u, and applying the horn-ruler, the field or strip of land is reduced to the three trapeziums A a b D, a c d b, and c n m d. Hence, by the scale planned from, we have A b = 246, a f = 162, b c = 265, a g = 174, d n = 275, and c i = 156; consequently, $\frac{A\,b \times a\,f}{2} + \frac{b\,c \times a\,g}{2} + \frac{d\,n \times c\,i}{2} = \frac{246 \times 162}{2} + \frac{265 \times 174}{2} + \frac{275 \times 156}{2} = 19926 + 23055 + 21450 =$ 64431 square links, equal 2 roods and 23 perches, the area required.

METHOD 2.

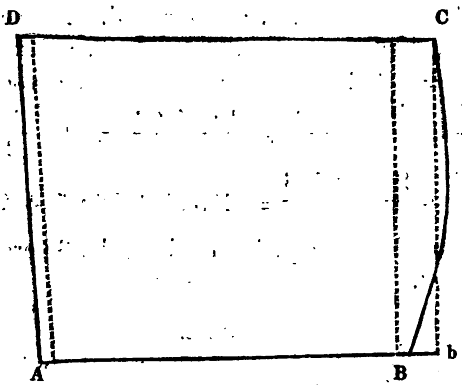
Take the perpendicular breadth at each end, also the length o p q w along the middle, by the scale plotted from; then, by the nature of trapezoids, we have $\frac{A\,D + B\,C}{2} \times \overline{o\,p + p\,q + q\,u} = \frac{88 + 90}{2} \times \overline{212 + 245 + 267} =$ 89 × 724 = 64436 square links, equal 2 roods and 23 perches, the same area as before, nearly.

Note.—The second method is chiefly used in ascertaining the quantity of a ridge of land, in open township fields; and will answer the same purpose when the quantity is required, without planning.

Note.—The area of a ridge that tapers to a point, is likewise found by the same method.

EXAMPLE 5.

Suppose A B C D to represent a flat of open field land, planned from a scale of 4 chains to an inch, containing sixteen ridges of equal breadth, whose lines of partition, or furrows that part the ridges, lay in the direction of A B or D C; required the area of the whole flat, and also that of each ridge contained therein.



First, by proceeding with the horn and parallel rulers, the figure A B C D is reduced to the trapezium A b C D, whose area I find to be 589835 square links; and the perpendicular breadth of the ends A D and B C = 680 and 666. Hence, then, $\frac{680}{16} = 42.5$ links, the perpen-

dicular breadth of each ridge at the end A D, and $\frac{666}{16}$ = 41.625 the breadth near the end B C; consequently, $\frac{42.5 + 41.625}{2} = \frac{84.125}{2} = 42.06$ the mean breadth; and, marking off the above breadths at or near each end, and measuring the length of each ridge along the middle of it, beginning at that adjoining the side A B, the operation will stand as follows:—

	Lengths.		Areas.
No. 1, or 1st Ridge	= 794	Multiplied by 42.06.	= 33395
2 ditto	= 810		= 34068
3 ditto	= 826		= 34741
4 ditto	= 840		= 35330
5 ditto	= 854		= 35919
6 ditto	= 865		= 36381
7 ditto	= 876		= 36844
8 ditto	= 880		= 37012
9 ditto	= 885		= 37223
10 ditto	= 890		= 37433
11 ditto	= 896		= 37685
12 ditto	= 900		= 37854
13 ditto	= 896		= 37685
14 ditto	= 896		= 37685
15 ditto	= 896		= 37685
16 ditto	= 900		= 37854

The area of the flat = 584794 square links; which differing from the true area found above by 5041 square links, say, as the false area of all the ridges in the flat, is to the true area of the flat; so is the false area of every particular ridge, to the true area of every particular ridge. Consequently, the false area of every ridge, multiplied by the quotient arising from the true area of the flat, divided by the false area, gives the true area. Hence, then,

	False Areas.		True Areas.
No. 1, or 1st Ridge	= 33395	Multiplied by 1.007 = $\frac{589835}{584794}$ nearly.	33628
2 ditto	= 34068		34306
3 ditto	= 34741		34984
4 ditto	= 35330		35577
5 ditto	= 35919		36170
6 ditto	= 36381		36635
7 ditto	= 36844		37101
8 ditto	= 37012		37271
9 ditto	= 37223		37483
10 ditto	= 37433		37695
11 ditto	= 37685		37953
12 ditto	= 37854		38118
13 ditto	= 37685		37953
14 ditto	= 37685		37953
15 ditto	= 37685		37953
16 ditto	= 37854		38118

Their sum = 588898 square links; which being so near the true area first found is then taken for the area of the flat; and the proprietor of each ridge being known, the quantity each proprietor has in the flat, is then readily known; and, in case of an inclosure, the account thereof makes that part of the flat book from whence each person's property is collected and carried into the property book.

Note.—The above method is what I hinted at before, respecting flats of open field land, whose contained ridges bear a proportionate breadth one to another.

Note 2.—In surveying for a valuation only, all the ridges in the field must be regularly described, numbered, and entered, under the names of the occupier, in the survey-book, the same as fields of old inclosure.

ar
t;
y
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s

ate. 2.

A PLAN,

of

Kirkdale Farm,

E

FIELD-BOOK

Begin.^{g} of Field Bk. to Plan 2nd

	00	200		
	27	145		
	31	100		
⊙1 North	21	00		
		1606		△ 84 Links East of ⊙3
		1080	64	
		900	38	
		720	24	
		508	22	pas.^{d} +st
		340	25	
⊙8		55	48	++st
		1054		△ 68 Links S. of ⊙9
		995		++st
	27	927		
⊙2 North	22	300		
		3083		△ 15 Links West of ⊙1
		2380		⊙10. a.
		1850		⊙10
		1829		++
	36	1750		to +
	27	1730		pas.^{d} +
		1694		++
		1600		⊙9
		1019		++
⊙6 South		950		⊙8
		2400		⊙7
		2300	75	⊙6 begun
		2180		++ of Wood
		1750		+ R^{d}
		1660		⊙5
		1630		++st at 7
		1620	15	pas.^{d} +
		1470	54	
		1090	90	
		870	75	
		800		⊙4. a.
		500	50	
⊙4 North		150	10	
		1973	40	⊙4
		1890	75	
		1770	60	
		1610	55	
		1586	42	
		1550		⊙3
		1510	36	
		1490	30	++
		1440	42	
		1280	50-5	
		930		⊙2
		911		++
		660	24	
		530	25	
		340	32	
		265	35	
⊙1 East		00	40	

Plan 2

		1481		△ 10 Links North of ⊙7
		1400		ent.d Wood
		1225		+d Road
		1040		⊙15
	54	1013		pas.d
	47	738		
	53	700		
	29	570		
		520		+ +
		500		⊙14
		430		+ +
⊙13 East		200	68	
		1800		⊙13
		1673		+ +
	78	1640		
	64	1590		
	68	1550		
	56	1460		
	61	1430		
	58	1385		
	45	1190		
	47	1160		
	40	1100		
	32	962		
	16	700		⊙12
	14	560		
	00	480	00	
		430	3	
	00	400	00	
	12	340		
	13	250		
	26	160		
	40	115		
⊙ 9 North	40	100		
⊙11		600		⊙10. a.
		789		⊙10
	10	730		
	5	550		
	12	500		
	13	440		
	00	80		
⊙11	17	00		
	7	940		⊙11
	9	870		
	8	800		
	9	750		
	2	600		
	15	570		
	30	356		
	00	250		

FIELD-BOOK

Plan 2

⊙ 19 West		402		⊙ 4. a.	
	40-12	953		△ 8. Links E. ⊙ 4 (to ┐ 12	
	16	865			
	17	820			
	11	680			
	20	450			
	32	325			
⊙ 19	27	160			
		874		⊙ 19. at +	
	18	820			
	24	775			
	28	650			
	38	555			
	47	390			
⊙ 18	26	270			
		259		⊙ 5	
	32	200			
	10+40	60		⊙ 18. pas.d +	
⊙ 17 West	30	00			
	13	700		⊙ 17.	
	24	500			
⊙ 7 South	27	380			
⊙ 16. a.		396		⊙ 15.	
		664		△ 127. Links North of ⊙ 7	
	20	620			
	35	585			
	40	500			
		445		⊙ 16. a.	
	52	400			
	73	200			
		150		+ + of Wood	
	90	20		to <	
⊙ 16	30	00		to + of Wood	
	48	650		⊙ 16	
	30	584			
	33	554			
	23	515			
	14	440			
	18	420			
	12	380			
	6	300			
⊙ 15	6	150			
		1158		△ 13 Links East of ⊙ 14	
		500	31		
⊙ 8 North	50	200	47	+ ÷ to	

TOM

SECTION VI.

OF SURVEYING AND PLANNING A FARM, TOWNSHIP, OR LARGE ESTATE, WITH THE METHOD OF KEEPING THE BOOKS, &c. USED IN THE VALUATION.

By surveying and planning a farm, township, or large estate, is meant, the taking of the necessary dimensions, &c. upon the ground, so as to form the plan or figure of the whole estate upon paper; as well as the figure of each separate field, &c. contained therein. This, performed well, is the principal aim of the new method of surveying.

PROBLEM I.

To survey and plan the small Farm, (see Plate,) being the late J. B.'s Kirkdale Farm.

METHOD OF SURVEYING.

It is evident, that the readiest way of measuring this farm, would be to inscribe in it a large triangle. Measure then from any corner, as from ⊙ 1 Eastwards to ⊙ 4, for a base line; make the intermediate ⊙ 2 and 3; and enter the hedges crossed, offsets taken, &c. in your field book, in their proper places, as before directed. Next measure from ⊙ 4 Northward, to ⊙ 7, and make ⊙ 4 a, 5, and 6. From ⊙ 6 measure Southwards, to a △ 15 links West of ⊙ 1, and make ⊙s. 8, 9, 10, and 10 a. Then have you got all the three sides of the triangle. Next, measure from ⊙ 2 to the △ 68 links South of ⊙ 9, and from ⊙ 8 to the △ 84

ate 4

⊙ [illegible]

down ⊙ 7, & ⊙; then is the first triangle constructed:

FIELD-BOOK

Begin^g of Field Bk. to Plan 4^th.

		420	60		
		300		+÷	
	10	284		pas^d +	
⊙18. Westwards		243		⊙19	
		4420		⊙18	
	60	3830		pas^d ÷	
		3818		⊙17	
		3790		+÷	
		3750	60	to >	
		3560		+÷	
	58	3190		pas^d +	
		3135		⊙16	
		2840		+÷	
		2710	30		
		2600	40		
		2520		⊙15	
		2480		+÷	
		2340	40		
		2060		⊙14	
		2018		+÷	
		1910	90		
		1733	110		
		1500	93		
		1340		+÷	
		1320		⊙13	
		1285	90		
		904	80		
		635	40	⊙12	
△665.(S^t of ⊙1 Produced,) Eastw^d.		593		+ +	
		4440		⊙11	
		4400		++	
		3480		⊙10	
		3405		++	
		3164		⊙9	
		3126		++	
		2940		⊙8	
		2904		++	
		2740		⊙7	
		2716		++	
		2480		+÷st	
		2314		+÷st	
		2285		⊙6	
		2180		+÷	
		2140		⊙5	Lane
		2100		+÷	
		1635		+÷	
		1610		⊙4	
		1130		⊙3	
		1080		++	
		390		+÷	
⊙1. North for a Base Line		360		⊙2	

		875		⊙30	
	20	860		pas^d +	
		355		+ ÷	
	30	545		to ÷	
		530		⊙29	
	28	400			
	45	270			
	40	165			
⊙28	55	35		pas^d ∓	
		2662		⊙18	
		2313		+ ÷	left off
		1905	90		begun
		1835		⊙28	
		1780		+ +	
		1730		⊙27	
	46	1645			left off
	40	1515			
	50	1255			
	45	1050	45	left H. (begun)	
		1005		+ ÷	
		975		⊙26	lane
		910		+ ÷ at + or 7	
	15	785			
	20	480			
		434		+ +	
⊙10		380		⊙25	
		1190		⊙10	
	60	1030			
		700		⊙24	
	45	680		pas^d ÷	
	40	190		pas^d +	
		180		⊙23	
⊙22	50	150			
		3625		△ 550 L. prod. N. from ⊙11	
		2535		⊙22	
		2425		+ +	left off
		2400	45		
		2350	116		
		1995	14		
		1930		+ + at +	
		1900		⊙21 (lane	
		1855		+ ÷ at ÷	
	15	1750			
	30	1645			
	90	1600			
	15	1190			
	10	1080			
	30	1045		pas^d +	
		1010		⊙20	
		930		+ ÷	
		645	20		

FIELD - BOOK

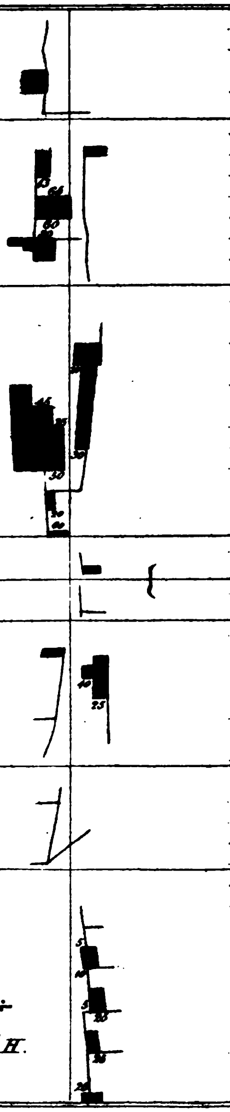
Plan 4

		490	15	ent.d H. & pas.d	
		445	24	left H. & pas.d	
		390	30	ent.d H.	
		380		⊙ 36	
		360	15+30	left H. & pas.d +	
		326	25-15		
		280	30	ent.d H.	
		265		⊙ 35	
		248	15+20	left H. & pas.d +	
		170	40-20	pas.d + & ent.d H.	
		140		⊙ 34	
⊙ 26		50	55		
		830		⊙ 33	
	25	716			
	30	460			
	30	265			
⊙ 24	20	110			
		770		⊙ 11	
	36	578			
	37	300			
⊙ 28	40	80			
		690		⊙ 23	
	20	510			
⊙ 31	30	210			
		875		⊙ 30	
		810	30		
		390	30		
⊙ 32		60	30		
		1236		⊙ 26	
	40+20	1180		pas.d ∓	
	10	1035			
		985	65		
	15	900			
		833	55		
	18	785			
		750		⊙ 33	
		710	50	pas.d +	
	10	435	58	pas.d +	
		410		⊙ 32	
		232	55	pas.d +	
		205		⊙ 31	
	25	125			
⊙ 21		50	40		
		920		⊙ 17	
	35	700			
	40	570			
	46	470			
⊙ 29	95	265			
		1270		⊙ 20	
	28	1200			
	22	1100			

FIELD BOOK

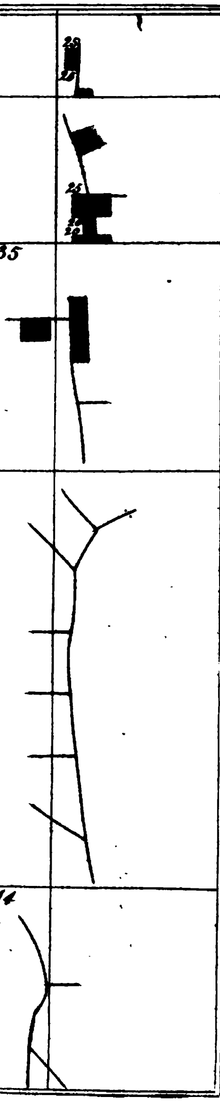
Plan 4

	68	340		
	60	300		
	80	100		left H.
	80	70		ent^d. H.
△ 40 Links S. ⊙ 38	75	30		+ +
		435		⊙ 37
		374	15	ent^d. H.
	70-15	350		ent^d. H.
	65+0	320	15	left H.
	60-60	290	14	ent^d. H.
	30+30	225	20	+ +
	30	140		ent^d. H.
⊙ 7		90	18	
		572		△ 20 L. S. ⊙ 8
		470	60	
		380	10	left H.
		350	30-20	
	95	325		left H.
	40+45	290		
	20+25	270		
		260	45-30	ent^d. H.
	70-50	235		ent^d. H.
		170	36	+ + & left H.
	70-20	130		ent^d. H.
⊙ 36	60+10	60		left H.
		180		⊙ 34
⊙ 41		115	30	ent^d. H.
		125		⊙ 41
⊙ 40		100	20	pas^d. +
		590		⊙ 35
	20	535		ent^d. H.
		520	60-40	
		500	95-25	ent^d. H.
	26	436		pas^d. ÷
		416		⊙ 40
⊙ 39	40	200		
		410		⊙ 8
	31	348		pas^d. +
		255		⊙ 39
		230		+ ÷
⊙ 25	37	210		pas^d. ÷
		1015		⊙ 5
		895		⊙ 38
		880	35	pas^d. ÷
		825	36+5	left H.
		760	40÷10	ent^d. H. & pas^d. +
		740	43	left H.
		665	35+5	ent^d. H. & pas^d. ÷
		630	35	left H.
		575	35	pas^d. ÷ & ent^d. H.
		560		⊙ 37
		520	40+20	left H.



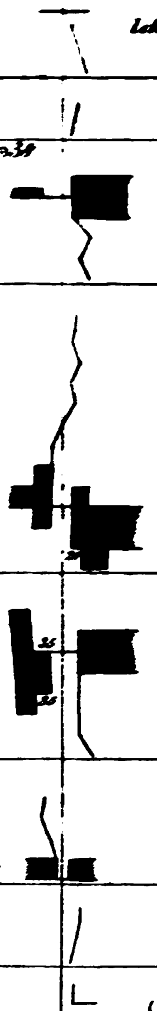
Plan 4

		373		⊙52
		254	17+25	left H.
		178	40-25	ent.^{d} H.
⊙51		100	38	left H.
		295		⊙26
		248	36	left H.
		186	45	ent.^{d} H.
		155		⊙51
		125	25+25	left H. & pas.^{d} +
		94	40-20	
⊙50		40	20+20	
		838		△ 30 Links S. of ⊙35
		785		⊙50
		770	15	left H.
	20	754		+ ÷ & left H.
	20	720		ent.^{d} H.
		690	15	ent.^{d} H.
		550	20	
		440	25	pas.^{d} ÷
		420		⊙49
		205	28	
⊙48		90	30	
		1100		⊙27
		1000	30	
		955	86	pas.^{d} ∓
		810		+ ÷
		800	18	to ÷^{s}
		780		⊙48
		650	20	
		625		+ ÷
		605		⊙47
		540	15	
		525		+ ÷
		490		⊙46
		420		⊙45
		390		+ ÷
		370	30	
		295	35	
		225		+ ÷
		205	50	to ÷
		185		⊙44
△130. L. W. of ⊙43		86	66	
		1805		△120 Links S. ⊙14
	65	1575		
	30	1325		
		1160		⊙43
	10	1110		to ÷^{s} & + ÷
	30	850		
		608		⊙42
	45	595		to ÷^{s}
		570		+ ÷



FIELD BOOK

	20	560			began
		495		⊙ 43	
		448		+ ÷	
		370	10		Left off
		300	25		
⊙ 3		140	30		
		415		⊙ 4	
		310	15		
⊙ 42.		230	10		
		946		△110 Links N. of ⊙34	
		750	4	left H	
		640		+ ÷	
		470	10	ent^{d} H	
		580	40		
		360	33		
⊙ 44.		145	40		
		920		⊙ 45	
		815	15		
		590	10		
		475	20		
		370	10		
		315	15		
		305		+ ÷	
	15	295			
	15	174		left H.	
		154	10	left H.	
		115		+ ÷	
	20	90		ent^{d} H.	
		70	30-20		
△120 Links S. of ⊙37		50	30	ent^{d} H.	
		909		⊙37.	
	60	826		left H.	
		800	10	left H.	
	35+25	794		+ ÷	
		775	10	ent^{d} H.	
	60-35	765			
	60	740		ent^{d} H.	
		535	14		
⊙ 46.		358	30		
		880		⊙ 47.	
	26	760			
	30	525			
	20	240			
	10	130	10	left H^{s}	
△90 Links S. of ⊙36.	10	100	10	+ ÷ & ent^{d} H^{s}	
		980		⊙ 13.	
		730	35		
		390	30		
△55 Links N. of ⊙48		84	13		
		170		⊙ 49.	
⊙ 52		45	20	pas^{d} +	



		1.50		⊙ 4	
		280	10		
⊙ 60.		180	20		
		1.520		⊙ 3	
	20	1460	35		
	12	1.390	40		
		12.90	40		
		11.90		⊙ 60.	
		116.5	20	pas^d. +	
	37	11.55		pas^d. ÷	
		1120	18		
	40	1020			
	40	935			
		775	20		
	45	660		pas^d. ÷	
		650		⊙ 59.	
		540		⊙ 58.	
	60	185	00	at +^{s}	
		360	00		
	.50	200		pas^d. +	
		186		⊙ 57.	
		170	25		
⊙ 56.	.36	8.5	30		
		1860		⊙ 56.	
		1825		+ +	lane
	60	177.5			
	.50	148.5		ent^d. Gate.	
	70	107.5			
	00	.985		+ + at ÷^{s}	
	108	960		to <	
		.9.50		⊙ 55.	
	90	874		to Gate.	
	33	460			
	00	290		÷ at ÷^{s}	
		2.50		⊙ 54.	
	36	.90		ent^d. Gate.	
⊙ 1. Westwards.	45	00			
		440		⊙ 12.	
		420	40	to Gate.	
		275	15		
		70	45		
⊙ 1.		40	45		
		560		⊙ 2.	
		410	15		
		300	20		
		250	40		
⊙ 53.		15	45	pas^d. ÷	
		695		⊙ 53.	
		505	55		
⊙ 43.		285	60		
		1150		⊙ 13.	
	20	812			

FIELD-BOOK

Plan 4

	90	900		⊙73. to +^{g}	
		356		+ +	
		466	30	pas.^{d} +	
		436		+ +	
⊙10		390		⊙72	
		3604		△230. L. N. of ⊙11 prod.	
		3330		⊙71	
		3290		+ +	
	50	3254		to ∠	
	20	2795			
	10	2720			
		2710		+ +	begun
		2680		⊙70	
		1970		⊙69	
		1765	30		left off
		1630		+ ÷	
		1520		⊙68	
		830		⊙67	
		770		+ ÷ at +^{g}	left off
	80	690			
	40	555			
		355		+ +	
●66		105	50		
		1900		⊙66	
		1820	10	pas.^{d} +	
		1635	30	ent.^{d} Gate	
		1380	50	pas.^{d} ÷	
		1360		⊙65	
		1305		+ ÷	begun
		1260		⊙64	
		910		⊙63	
⊙56. Westwards		25		+ ÷	
		705		⊙58	
	48	560			
	58	385			
	48	225			
⊙62	50	135			
		254		⊙54	
		190	28		
⊙2		95	20		
		275		⊙3	
⊙61		160	40		
		1135		⊙55	
		850	16		
		725	5		
		670	00	pas.^{d} ÷	
		630	00	⊙62	
		404	10	+ ÷	
		372		⊙61	
		246	30		
⊙4		40	45	pas.^{d} ∓	

Plan 4

		450		⊙81	
		400	40		
		333	40		
		245	50		
		205	44		
		140	20	pas.^{d} ∓	
⊙69		70		+ +	
		1130		⊙67	
	40	1054			
	30	710		pas.^{d} +	
		660		⊙80	
		528		+ +	
		480		⊙79	
	40	244			
△130 L. S. of ⊙ 74	45	30		pas.^{d} ∓	
	40	680		⊙69	
⊙78	40	280			
		955		△45 L. S. of ⊙72	
		640	78		
		370	96	pas.^{d} +	
		336		⊙78	
		240		+ +	
△35 Links S. of ⊙70	80	125			
		316		⊙72	
	28	250			
		184		+ +	
⊙9	28	50			
		500		⊙77	
	25	335		pas.^{d} +st	
⊙8	28	175		pas.^{d} +st	
		960		⊙6	
	20	854			
	30	530		pas.^{d} +	
		500		⊙77	
	20	370			
△30 L. S. ⊙73	16	230			
		2585		⊙63	
		2395	60		
		2315		⊙76	
		2280		+ +	
		2170	70		
		1900	60		
		1800		⊙75	
		1770		+ +	
		1740	45		
		1600	40		begun
		1440		+ +	
	80	1372			begun
		1272		⊙74	
	135	1100			left off
		940		+ +	

⊙83		195		△240 Links S. of ⊙68
		325		⊙68
	25	266		
⊙83	40	190		
	25	470		⊙83 to <
⊙67	35	154		
		260		⊙63
⊙82	30	160		
		350		⊙64
	15	228		to Gate
		20		+ ÷
⊙82	35	10		to +
	20	1005		⊙82
	30	765		entd. Gate
	20	476		
	20	371		
	10	95		
△50L. prod. S. from ⊙56	20	30		to + or <
		900		⊙57
	40	535		
⊙76	35	140		
		955		⊙75
		580	40	
		435	35	
⊙59		70	15	
		1020		⊙60
		865	35	
		500	30	
		340	30	
		200	40	pasd. ∓
⊙74		160	40	+ ÷ to ÷
		950		⊙65
		760	10	
		575	00	
		460	30	
⊙80		200	30	
		770		⊙73
		565	20	
		340	15	
⊙81		180	20	
		945		⊙79
		765	45	
		528	10	
		490		+ +

Next lay down the intermediate ⊙s in every line or side of the triangle; after that the interior lines, offsets, &c. by the above quoted Problem, until you have completed all the work within the triangle. The other triangles, constructed upon the sides of the first, you must lay down in the same manner, one after another, as entered in your field book, until you have completed the plan. When this is done, in small surveys like the present, after you have inked the fences, &c. rub out the chain lines, and number the fields; then find the area of each field as directed in Section 5, which quantity put opposite its number and name, as in the Plan Plate; write round the plan the proprietors' names adjoining, add the title to it, &c., and it is done.

PROBLEM 2.

To survey and plan the Estate or Township of B, in the County of S——t. (See Plan.)

METHOD OF SURVEYING.

First, find a particular object or mark, in the direction of which to measure, for a base line, as before directed. Then make ⊙ 1; but observe to make it in such a place, that the line measured from it, in the direction of such object, will have the following conveniences, *viz.* that it will intersect as few woods, waters, &c. as possible, and that you can see your object or mark from any part. Measure from thence, with the greatest exactness, to ⊙ 11; make and note down in your book, likewise, the intermediate ⊙s all along, where you think necessary; also note down every hedge, brook, road, &c. you cross, as well as every offset taken in the line, as directed in Section 4. Then go back to ⊙ 1, and produce the base line Southwards to △ a. Next measure from △ a, along the East side of the Estate, in a Northerly direction to ⊙ 18, and from ⊙ 18 measure

Westwards along the North side of the Estate, until you cross the base line at △ b, produced from ⊙ 11; make and note down ⊙^{s}, enter remarks, offsets, &c. in both lines, the same as in the base line; then is that part of the estate, laying on the North side of the base line, circumscribed by a triangle. Next measure the proof lines, *i. e.* from one fixed point or ⊙ to another; make ⊙^{s} and enter every thing necessary in them, as in the other lines, until you have traced out every hedge, building, &c. within the circumscribing triangle; that is, from ⊙ 2 measure to ⊙ 10, and make ⊙ 23 and ⊙ 24 between; then from ⊙ 11 to ⊙ 23, from ⊙ 10 to ⊙ 16, from ⊙ 28 to ⊙ 20, from ⊙ 29 to ⊙ 17, from ⊙ 21 to ⊙ 26, &c. observing always to finish one piece taken off within the circumscribing triangle, before you take off another. Thus, when you have measured from ⊙ 22 to ⊙ 10, measure from ⊙ 23 to ⊙ 11, before you measure from ⊙ 10 to ⊙ 16, lest you should forget to trace any hedges, &c. and so commit an error. After finishing every thing within the triangle circumscribing the North side of the estate, return to ⊙ 1, and measure from it Westwards, or along the South side to ⊙ 56; from ⊙ 56 measure Northwards to ⊙ 5, in the base line; make ⊙^{s}, and note down every thing worthy of remark in measuring them, as before; then have you a small triangle taken off on the other side of the base line, which triangle complete as above, before you leave it. Next measure from ⊙ 56 Westwards to ⊙ 66, and from ⊙ 66 Northwards to △ b; completing all the inside work of this last piece, or trapezium, taken in. After that, proceed to take in all the small triangles on the outside of the great or circumscribing lines, *viz.* measure from the △ produced Southwards, from ⊙ 56 to ⊙ 82, and from ⊙ 82 to ⊙ 64; likewise from ⊙ 67 to ⊙ 83; and from ⊙ 83 to ⊙ 68; then is the South side of the base line completed, as well as the whole of the estate surveyed.

Note.—If it happens, in surveying a township, that the town contained in it is so large, that a straight line cannot be got through it, measure and plan

all the land that lies round it first; then begin at a known point or ⊙, in some of the principal lanes that lead to the town, and trace it, as well as the streets it leads into. Note down all the front houses, &c. also take the angles at every turn, as well as the angle one street makes with another, until you arrive at some other fixed point or ⊙, in the land surveyed. Proceed thus with the rest of the lanes and streets, until you have finished them all. After you have planned them, take a sketch of the streets, &c. from the plan, and measure, with your offset-staff, the yards, gardens, back houses, &c., noting the dimensions upon your sketch, as you take them, with the figure, &c. measured. Lastly, transfer from your sketch, the said figures to the plan, and it is done.

If a large town or city be required to be surveyed alone, it is done exactly in the same manner, *viz.* by first tracing the great streets, and afterwards the small ones, &c. But the surveyor, for such a piece of work, should be provided with a chain made on purpose, of about 40 feet long, divided into links of one foot each, so that in planning from a large scale, as one chain to an inch or more, an angle or any remark whatever may be accurately laid down on the plan, &c.

Note 2.—By returning back to the beginning of the base line, it is to be observed, that some advantage is had, *viz.* the marks or objects, in the direction of which we have to measure, are more readily obtained, than by measuring from the other end; otherwise we might have measured Eastwards from △ b to ⊙ 18, and from ⊙ 18 Southwards to ⊙ 1, as it makes no difference which end of the base line we begin at, provided we can only obtain proper objects to measure in the direction of, &c.

Note 3.—The South side of this estate might have been circumscribed, the same as the North side, with one triangle only. But, in surveying a large estate, whose boundary is very extensive and irregular, I make it a general rule, after measuring the base line, to constitute a triangle at the end of it, as in this Problem, or Problem 9, Section 4. Then from some known or fixed point, in the side of the triangle adjoining the land unsurveyed, as from ⊙ 56 in this case, I measure again Westwards, in the direction of some fixed mark at a distance, as far as I think necessary, as to ⊙ 66; from ⊙ 66 I measure up to the base line, &c. always dropping that line which runs on the outside of the estate, when I think I am going to measure what is unnecessary. But it is to be observed, in surveying, when either of the parts or sides of an estate (made by a base line measured through it) can be circumscribed with one triangle, the most accurate method will be to circumscribe it; although the above method, when carefully attended to by a practical surveyor, will differ very little from the truth. The only difficulty attending it is, in determining the intersection of the lines that form the triangles and trapeziums, constituted on the base line; for by determin-

ing them wrong upon the plan, the proof lines will not fit their proper places; and in time this will cause a multiplicity of errors, which it will be almost impossible to rectify, without the following or some such method be employed.

METHOD.

Should any of the lines on the plan be found not to agree, in the plotting, with their lengths measured in the field, in any triangle or trapezium, and the cause cannot be discovered easily, lay down all the lines contained in the trapezium, &c. last taken, as accurately as you can. Then, at a small distance from the said trapezium, on the base line, make a ⊙, from whence measure a line, perpendicular to the base line, to the outside of the estate; make ⊙s upon it in their proper places, complete all the lines between the perpendicular and trapezium before measured, then is the error corrected, provided the base line be rightly measured and laid upon the plan, as well as the true position of the perpendicular determined. For the surveyor may then begin at any ⊙, in or near the extremity of the said perpendicular, and measure along the outside of the estate, as far as necessary; after that, up to the base line; and so complete all the work within, as before directed—without having any thing more to do with the erroneous lines—until he has completed the survey on that side of the base line.

Note.—Some surveyors use parallel lines, or perpendiculars erected upon the base line, in order to obtain the plan of an estate; which, if rightly applied, will do in some cases; as when a quantity of land is intersected by woods, that cannot be well crossed or passed with the chain, at the outside of the estate, in one direct line. In that case, by erecting perpendiculars, or measuring lines at right angles to the base line in proper places, to the outside of the estate, making ⊙s, &c. in every line, they will answer the purpose wanted for the survey; for all of them being fixed lines, extending from the base line along the sides, and between the woods, triangles and trapeziums may be constructed upon them, ⊙s, &c. made, fences traced, both interior and those that are boundary fences. The boundary fences of the woods, on the outside of the estate, if very crooked, are got by fixing marks in such places as are most convenient, measuring the distance between those marks,

taking the offsets, and likewise all the angles, beginning at a fixed point, in a fast line on one side of the wood, and proceeding to some point in another on the contrary side; and in that manner is the plan of the estate obtained. Some surveyors likewise measure lines, in convenient places, from the base line, and determine their true position by an angular instrument, such as a circumferentor, &c.; some by a pocket compass—taking it for granted that the variation of the needle is the same in all parts of this island; a method which I think is not to be depended upon for correctness, both on account of the above variation being different in different parts, as well as the variation of variation, or diurnal variation, so long known to mathematicians. But the above method I have practised above thirty years, and always found it effectual, when properly applied, let the size or figure of the estate be what it might.

METHOD OF PLOTTING.

This estate, or any other, is plotted exactly in the same manner as the last Problem, or Problem 9, Section 4; due regard being had to the marks and characters entered in the field book; and of course it would be superfluous to give any directions here.

SCHOLIUM.

The difficulty in surveying and planning any large estate, is the making of ⊙s., the taking of offsets, the crossing and passing of fences, and other particular objects, that have to be laid upon the plan, in their proper places, and entering them correctly into the field book; for, after these are properly entered, the reader, by duly observing what I have here laid down, and attending to his field book, will, without any difficulty, be able to plan an estate of any size and figure.

The estate being thus surveyed and planned, next rub out the chain lines, ⊙s., &c.; then number the fields, beginning at a corner of the plan, with No. 1, and proceeding, number after number, until you have numbered every field, yard, &c. in the plan. *(See part of the Plan, Plate 6.)* Next write round the plan, the townships it is bounded

by, &c.; after that find a meridian line,* as near as you can, in some part of the estate, and transfer it to a corner of the plan, and fix the compass, or some part of it, thereon, to shew the four cardinal points. Then make a small book, called the book of particulars, and rule it into four columns, wherein enter opposite each number, beginning at number one, the tenant it belongs to, the name of the field or flat in which it is situate, and in what course of husbandry it then is, *viz.* whether arable, meadow, or pasture, in the following manner:—

* A meridian line for a field or estate, may be found nearly enough to answer the purpose, by fixing a pole perpendicularly in a station made in any line, and finding the true position of its shadow with the said line, at twelve o'clock, or when the sun is upon the meridian. Then, by transferring the shadow to the plan, a line parallel to it may be drawn in any place, which best suits the purpose.

SPECIMEN OF

THE BOOK OF PARTICULARS.

BOOK OF PARTICULARS.

No. Pn.	*Tenants' Names.*	*Names of Fields.*	*Course of Husbandry.*
1	John Harrison	Moor	Meadow.
2	Ditto	Town end Field	Pasture.
3	Ditto	Low Garth	Meadow.
4	James Shylock	Gill Bank	Arable.
5	Ditto	Middle Ditto	Ditto.
6	Ditto	High Ditto	Ditto.
7	Ditto	Ditto Ditto	Meadow.
8	Ditto	Great Flat	Pasture.
9	Ditto	Back Close	Meadow.
10	Ditto	House and Garth	Pasture.
11	Thomas Jones	Ditto	Ditto.
12	Peter Rose	Ditto	Ditto.
13	John Jackson	Ditto	Ditto.
14	John Harrison	Ditto	Ditto.
15	Thomas Horn	House and Garth	Pasture.
16	Ditto	Town end Close	Meadow.
17	Ditto	Far Ditto	Ditto.
18	Ditto	Low Ditto	Pasture.
19	Ditto	Moor	Ditto.
20	Ditto	Ditto	Arable.
21	John Blakelock	Mire-Dikes	Ditto.
22	Ditto	Ditto	Ditto.
23	Ditto	Ditto	Ditto.
24	Ditto	Ditto	Ditto.
25	Ditto	Back Garth	Meadow.
26	Thomas Garth	House and Garth	Pasture.
27	John Field	Ditto	Ditto.
28	John Blakelock	Ditto	Ditto.
29	Henry Horswill	Ditto	Ditto.
30	William Howard	Ditto	Ditto.
31	John Day	Ditto	Ditto.
32	Francis Tasker	Ditto	Ditto.
33	Ditto	Garth	Ditto.
34	Ditto	Back Close	Ditto.
35	Ditto	Moor Field	Meadow.
36	Ditto	High Close	Ditto.
37	Ditto	Moor End	Ditto.
38	John Blakelock	House Close	Ditto.
39	Ditto	Far Ditto	Ditto.
40	Ditto	Doe-Water	Pasture.
41	Ditto	Moor	Ditto.
42	Ditto	Ditto	Ditto.

Note.—Instead of the words arable, meadow, and pasture, their initials are frequently substituted, as will be shewn in the following Survey-Book.

The tenants' names, names of fields, &c. being thus taken down, according to the directions of some intelligent person living in the place or township surveyed; next, begin and calculate the area of each number, by one or other of the cases in Section 5; then select from the above book, each tenant's respective numbers, and enter them under his name, with the name of the field, course of husbandry, quantity, &c. into a book called the Survey-Book, whose leaves are divided into columns, proper to contain them, in the following manner, beginning with the tenants alphabetically, viz.:—In the first column, put the number of the field on the plan; in the second, its name; in the third, the course of husbandry it is then in; in the fourth, its quantity; in the fifth, its value per acre; in the sixth, its total value; and in the seventh, the course of husbandry it is to be occupied in, &c. The three last of these particulars must be left blank until the valuation takes place.

Note.—When taking dimensions from the plan, in order to calculate the area of every field, it is likewise necessary to have a small book, or a few sheets of paper, divided into columns, proper for the dimensions, &c. to be entered into; which book, or sheets, may be called the Book, or Sheets of Data; and it is formed thus—in the first column, is put the number; in the second, the base or diagonal of the figure; in the third, one half the perpendiculars; and in the fourth, the quantity.

THE

SURVEY BOOK

OF

THE TOWNSHIP OF B.,

IN THE COUNTY OF YORK.

BLAKELOCK, JOHN.

No. on the Plan.	Names of Fields.	Course of Husbandry.	Quantity.			Value per Acre.	TotalValue.		
			A.	R.	P.	SH.	£.	s.	d.
21	Mire Dikes	a	3	3	22	w w	2	2	9
22	Ditto	a	4	2	21	w o	3	0	2
23	Ditto	a	4	2	15	w n	3	4	4
24	Ditto	a	4	0	15	w s	3	1	5
25	Back Garth	m	2	3	19	w t	2	5	11
28	House and Garth	p	0	2	17	c e	0	12	2
38	House Close	m	5	2	38	w c	3	8	9
39	Far House Close	m	3	1	37	w n	2	8	9
40	Doe Water	p	3	2	27	w c	2	4	0
41	Moor	p	3	3	7	w w	2	1	9
42	Ditto.	p	5	1	33	w c	3	5	6
			42	3	11		27	14	9

= 554.750 shillings.

DAY, JOHN.

31	House and Garth....	p	0	2	23	w t.. t	0	10	7

= 10.583

FIELD, JOHN.

27	House and Garth....	p	0	3	2	c e	0	15	3

= 15.250

Course of Husbandry in which to be occupied.
To be continued.
Ditto.
To be sown with Seeds.
Ditto.
To be continued.
Ditto.
Ditto.
Ditto.
Ditto.
Ditto.
Ditto.

Having thus put down, under every tenant's name, the respective numbers, &c. in the survey book,* next copy a sketch, or reduced plan, from the original, (as directed elsewhere,) and number every field in it, with the same number as in the original; then is every thing ready for the valuer. Enter, then, from the valuer, the value per acre,† into the fifth column. Next find the total value of each number, and enter it in the sixth column; also add the value of every tenant's numbers into one sum, for the value of his farm; then have you completed the whole survey; except your employer wish to have a *clean plan and book*,‡ which must be copied from the original, attaching to each field, in the new plan, its respective number in the old one. Write round the estate, upon the plan, the names of the townships it adjoins upon, afterwards add the title, &c. (as in the Plates,) and the whole is completed.

* In very large surveys, where the farms are much altered or changed, it is necessary to have other two books, besides the above, after the estate is valued. In one are entered numerically all the numbers, one after another, writing opposite each number its present tenant's name, the name of the field, the quantity, the value per acre, the total value, and the new tenant's name; this is called the numerical book. The other is for the purpose of entering each tenant's new farm, as entered in the numerical book, making him Dr. for it, and giving him credit, at the same time, on the opposite side of the book, for the land taken from him; this is called the ledger.

† Every valuer has a key, unknown to any person but himself and the surveyor, by which he notes down the value per acre of each number. This key is a word of ten different letters. The key used in the preceding book is *wconstable*, which letters are substituted for figures in the following manner:—

KEY.

w	c	o	n	s	t	a	b	l	e.
1	2	3	4	5	6	7	8	9	0.

The word Cumberland, as well as various other words, may likewise be used, to prevent the value from being known, although the book may be seen.

‡ Since all colouring and shading of fields is disused, except the representing of woods, waters, and mountains, the chief art of making a neatly finished plan and book, depends on the *penmanship* of the artist; and as that is not a department of which I am treating here, I leave the reader to consult other authors on the subject, and shall proceed to lay down the method of dividing land.

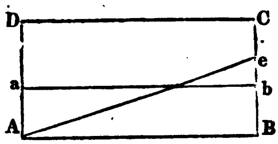
SECTION VII.

OF DETERMINING THE POSITION OF A RIGHT LINE, DRAWN FROM A GIVEN POINT, OR PARALLEL TO A GIVEN LINE, THAT WILL LAY OFF, FROM A FIELD OF ANY NUMBER OF VALUES, A PIECE OF LAND WORTH ANY PROPOSED NUMBER OF SHILLINGS, WHEN THE SAID LINE RUNS THROUGH ALL THE VALUES.

By the position of a right line is meant its exact situation, with respect to any other given right line or lines, as found by mathematical calculation.

PROBLEM 1.

At what distance, from the longest side of a parallelogram, must a right line be drawn parallel, to cut off from the said field, a piece of land, worth m *shillings, the length of the side being* l *links, and the value per acre of the field* n *shillings?*



SOLUTION.

Let A B C D represent a rectangular parallelogram, a b the line to be drawn parallel to A B, and A a the breadth to be laid off; for which put x. Then, since any quantity

multiplied by its value per acre, is equal to its value, we have, by Prob. 1, Section 2, Part 1, lx equal the area of the parallelogram A B b a, and $lx \times n = m$ its value, from whence is had $x = \frac{m}{ln}$ the distance required.

Cor. 1—Hence, if it be required to lay off from such field, a given quantity (q) of land, we have $x = \frac{q}{l}$, the breadth or distance required.

Cor. 2.—If it be required to lay off from a parallelogram, the same value, by a line drawn from the angle A to its opposite end B C, the perpendicular of the triangle will be $= \frac{2m}{ln}$; for if the perpendicular be denoted by x, we have $\frac{xl}{2} =$ the area ln, and $\frac{xln}{2}$ equal m its value, whence $x = \frac{2m}{ln}$.

EXAMPLE.

Suppose the length A B = 1500 links, and the value per acre of the field 20 shillings, required the breadth A a = B b, when the piece A B ba is worth 38 shillings.

$$\text{First } 1500 \times 20 = 30000 = l\,n.$$

Then $\text{A a} = \frac{m}{ln} = \frac{38.00000}{.30000} = 126.66$ links, the breadth required.

EXAMPLE 2.

Suppose A B = 1500 links, and the quantity to be laid off three acres, what is the breadth A a?

$$\text{A a} = \frac{q}{l} = \frac{300000}{1500} = 200 \text{ links, the breadth required.}$$

EXAMPLE 3.

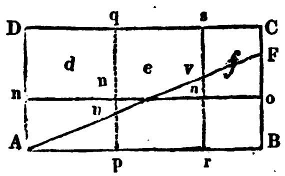
Suppose A B = 1500 links, required the distance B e, when the piece A B e is worth 38 shillings, the value per acre being 20.

$$\text{First, } 2m = 38.00000 \times 2 = 76.00000.$$
$$\text{And } l\,n = 1500 \times 20 = 30000.$$

Then $\text{B e} = \frac{2\,m}{l\,n} = \frac{76.00000}{.30000} = 253.33$ links, the distance required.

PROBLEM 2.

At what distance, from the longest side of a field in the form of a parallelogram, must a right line be drawn parallel, to cut off, from the said field, a piece of land worth m *shillings, when it consists of three values, whose quality lines lay parallel to its ends; the distance between them, on the side, being* a, b, *and* c *links, and their respective values per acre* d, e, *and* f *shillings.*



SOLUTION.

Let A B C D represent a rectangular field; p q and r s the quality lines; A p $= a$, p r $= b$, r B $= c$; $d, e,$ and f, the respective values per acre, and n o, the line to be drawn. Then, by putting $x =$ A n, the required distance, we have, by the preceding Problem, $a \times x \times d =$ the value of the parallelogram A p n n, and $b \times x \times e =$ the value of the parallelogram p r n n; also, $c \times x \times f =$ the value of the

parallelogram r'B o n. Consequently, $a\,d\,x + b\,e\,x + c\,f\,x = m$, the value to be laid off; from whence we obtain $x = \frac{m}{a\,d + b\,e + c\,f}$, the distance required.

Note.—Since the above method is general, let the number of values be what they will, I shall give the rule in words; it being that which is employed by every practical surveyor, in dividing a common, &c. among a number of proprietors, after laying off a piece first by guess, and, when necessary, making the quality lines parallel to each other, and to the ends, as in this Problem, as will be shewn hereafter.

RULE.

Multiply the length of each quality, by its respective value per acre, add them altogether, and divide the value or whole worth of the piece to be laid off, by the sum of them, and it will give the breadth required.

EXAMPLE.

Suppose A p, p r, and r B, to be 500, 400, and 300 links, and d, e, and f, 15, 10, and 8 shillings respectively; required the breadth A n, that will lay off the piece A B o n, worth 40 shillings.

$$\begin{aligned} \text{First, } a\,d &= 500 \times 15 = 7500 \\ b\,e &= 400 \times 10 = 4000 \\ c\,f &= 300 \times 8 = 2400 \\ & \qquad\qquad\quad \overline{13900} \end{aligned}$$

$$\text{Hence, A n} = \frac{m}{a\,d + b\,e + c\,f} = \frac{40{,}00000}{.13900} = 287.76$$

links, the breadth required.

Note.—It is here to be observed, that the Rule will hold good when the parallelogram is not rectangular, provided the distance x be always set off perpendicularly from the side or A B.

PROBLEM 3.

The figure, value per acre, and quality lines, being the same as in the last Problem, at what distance from the angle B, in the side B C, must a right line be drawn to the angle A, through all the values, to cut off a piece of land worth the same number of shillings.

SOLUTION.

Let A p $= a$, A r $= b$, $c =$ A B, and $x =$ B F; then, by similar triangles, $c : x :: b : \frac{b\,x}{c} = r\,v$; and $c : x :: a : \frac{a\,x}{c} = p\,u$: then, by Problem 2, Section 2, Part 1, $\frac{a\,x}{c} \times \frac{a}{2} \times d = \frac{d\,a^2\,x}{2\,c}$ = the value of A p u. And, by Problem 3, *ibid.* $\overline{\frac{b\,x}{c} + \frac{a\,x}{c}} \times \overline{\frac{b-a}{2}} \times e = \frac{b^2\,e\,x - a^2\,e\,x}{2\,c}$ = the value of p r v u. Also, $\overline{\frac{b\,x}{c} + \frac{x}{1}} \times \overline{\frac{c-b}{2}} \times f = \frac{f\,c^2\,x - f\,b^2\,x}{2\,c}$ = the value of r B F v. Hence the sum of the three values, $\frac{d\,a^2\,x + b^2\,e\,x - a^2\,e\,x + f\,c^2\,x - f\,b^2\,x}{2\,c.} = m$; from whence is found $x = \frac{2\,c\,m}{d\,a^2 + e\,b^2 - e\,a^2 + f\,c^2 - f\,b^2}$, the distance required.

EXAMPLE.

The numbers being the same as in the last example, we have $a = 500$, $b = 900$, $c = 1200$, $d = 15$, $e = 10$, $f = 8$, and $m = 40$; to find x ($=$ B F.)

$$\begin{aligned} \text{First, } a^2d &= 500\times500\times15 = 3750000\\ b^2e &= 900\times900\times10 = 8100000\\ c^2f &= 1200\times1200\times8 = 11520000\\ &\qquad\qquad\qquad\qquad\quad \overline{23370000} \end{aligned}$$

$$\begin{aligned} a^2e &= 500\times500\times10 = 2500000\\ b^2f &= 900\times900\times8 = \underline{6480000}\\ &\qquad\qquad\qquad\qquad\qquad 8980000 \end{aligned}$$

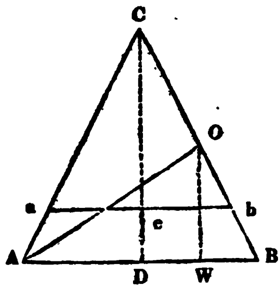
$$da^2+eb^2+fc^2-ea^2-fb^2 = 14390000$$

$2\,c\,m = 1200 \times 2 \times 40.00000 = 9600000000$; and

$\dfrac{2cm}{da^2+eb^2+fc^2-ea^2-fb^2} = \dfrac{960000}{1439} = 667$ links, the distance required.

PROBLEM 4.

At what perpendicular distance, from the base of a triangular field, must a right line be drawn parallel, to cut off, from the said field, a piece of land worth w *shillings, the value per acre of the field being* n *shillings, and its base and perpendicular each* l *links respectively.*



SOLUTION.

Let A B C represent the field, A B the base, D C the perpendicular, and a b the line to be drawn; then $\dfrac{AB \times DC}{2} = \dfrac{l^2}{2}$ is the area or quantity of land in the

whole field, by Problem 2, Part 1, Section 2; and, by proportion, we have $n : w :: a$ ($= 100000$ links) : $\frac{w\,a}{n}$ the quantity to be laid off, worth w. Hence $\frac{l^2}{2} - \frac{w\,a}{n} = \frac{l^2 n - 2\,w\,a}{2\,n}$ = the area of the triangle a C b. And, by Euclid, 20, 6, $\frac{l^2}{2} : \frac{l^2 n - 2\,w\,a}{2\,n} :: l^2 : \frac{l^2 - 2\,w\,a}{n}$ the square of the perpendicular C e; consequently $l - \sqrt{\frac{l^2 n - 2\,w\,a}{n}}$ will be the breadth required.

EXAMPLE.

Suppose the base and perpendicular each 900 links, and the value per acre of the field, 20 shillings; required the breadth e D, when the piece laid off, by the line a b, is worth 32 shillings.

First, $l^2 n = 900 \times 900 \times 20 = 16200000$

$2\,w\,a = 2 \times 100000 \times 32 = 6400000$

$$\frac{l^2 n - 2\,w\,a}{n} = \frac{9800000}{20} = 490000$$ and

$\sqrt{490000} = 700$. Hence, $900 - 700 = 200$ links, the breadth required.

PROBLEM 5.

The figure, dimensions, &c. remaining the same, (as in the last Problem,) at what distance from the base, in one of the sides, must a line be drawn to its opposite angle, to cut or part from the field a piece of land, worth the same number of shillings.

SOLUTION.

Let $WO = x$; then $\frac{l\,x}{2} \times n = w$; whence $l\,n\,x = 2\,w$, and $x = \frac{2\,w}{l\,n}$. Then by similar triangles $l : \frac{l}{2} :: x : \frac{x}{2} = BW$; consequently, by Euclid 47.1, $\sqrt{x^2 + \frac{x^2}{4}} = \frac{x}{2}\sqrt{5} = \frac{w}{l\,n}\sqrt{5} = OB$, the distance required.

EXAMPLE.

Required the distance O B, when the line O A, will lay off next the base, the same value as in last example.

First, $w\sqrt{5} = 32\sqrt{5} = 71.55424$ or 7155424 links,
$l\,n = 900 \times 20 = 1800$.

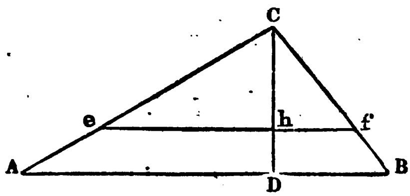
Then, $\frac{w\sqrt{5}}{l\,n} = \frac{7155424}{1800} = 397.5$ links, the distance required.

Note.—If it is required to divide a triangular field among two, three, or more persons, in any proportion, by lines drawn from an angular point to its opposite side; the said side must first be divided in proportion to their respective areas to be laid off. Thus, as the sum of all their areas (or the area of the whole field,) is to the base or side to be divided; so is each person's respective area, to his share of the base or side.—(*Euclid*, *Prop.* 1, 6.) Then from the extremity of each person's base, draw or stake out lines to the opposite angle of the triangle, and it is done.

Or, if it be required to divide the field by lines drawn from a point in the base or side; first find each person's share of the base as above directed; then draw a line from the given point to the vertical angle, and by Prob. 25, Part 1, Section 1, reduce the triangles to others of equal area, but of different bases; making an angle in each person's share come to the point given; from whence, having calculated the points where the divisions cut the sides of the field, draw or stake out the lines; and it is likewise done.

PROBLEM 6.

At what distance D h, *from the base, of a triangular field of two values, must a right line be drawn parallel to it, to cut off, from the field, a piece of land, worth* w *shillings; the quality line being the perpendicular let fall from the vertical angle upon the base, the length of which is* p *links, the segments of the base* b *and* c *links, and the value per acre* m *and* n *shillings.*



SOLUTION.

Let A B C represent the field, D C the perpendicular, e f the line to be drawn parallel to A B; and let D C be denoted by p, and h C by y; then $\frac{p\,b\,m}{2}$ = the value of A D C, and $\frac{p\,c\,n}{2}$ = the value of D C B; whence $\frac{p\,b\,m + p\,c\,n}{2}$ = the value of A B C, and $\frac{p\,b\,m + p\,c\,n - 2w}{2}$ = the value of e f C; consequently, by Euclid 20, 6, $\frac{p\,b\,m + p\,c\,n}{2} : \frac{p\,b\,m + p\,c\,n - 2w}{2} :: p^2 : y^2$; from whence, by reduction, is found, $y = \sqrt{\frac{p^2 b\,m + p^2 c\,n - 2w\,p}{b\,m + c\,n}}$.

Hence, $p - y$ = the distance on the perpendicular from the base, where the line must be drawn.

EXAMPLE.

Suppose the perpendicular $CD = 400$, the segments of the base A D and D B = 700 and 300 links, $m = 20$, $n = 10$, to find the distance D h, when the line e h f will cut off, next the base, a piece of land worth 20 shillings.

$$\left.\begin{array}{l}\text{First } p^2\,b\,m = 400 \times 400 \times 700 \times 20 = 2240000000 \\ \quad p^2\,c\,m = 400 \times 400 \times 300 \times 10 = 480000000 \\ \quad 2\,w\,p = 2 \times 20.00000 \times 400 \ = 1600000000\end{array}\right\}\text{ and}$$

$$b\,m = 700 \times 20 = 14000$$

$$c\,n = 300 \times 10 = \ 3000.\text{——Then,}$$

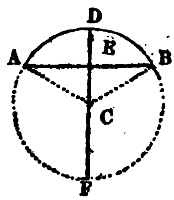
$$\sqrt{\frac{p^2 b m + p^2 c n - 2 w p}{b m + c n}} = \sqrt{\frac{2240000 + 480000 - 1600000}{14 + 3}}$$

$$= \sqrt{\frac{1120000}{17}} = \sqrt{65882} = 256.6. \quad \text{Hence, } 400 - 256.6 =$$

143.4 links, the distance required.

PROBLEM 7.

At what distance from the extremity of the diameter of a circular field, must the chord be drawn, at right angles, so that its respective sector will be worth w *shillings; the diameter being* d *chains, and the value per acre of the field* n *shillings.*



SOLUTION.

Let A D B F A be the circular field, $d = DF$, and $a = .7854$, then by Problem 12, Section 2, Part 1, we have

$d^2 a =$ the area of the whole circle, and $n : l :: w : \frac{w}{n}$ the area of the sector A D B C A; and by Rule 2, Problem 11, *ibid.*, $d^2 a : 360\ (r) :: \frac{w}{n} : \frac{r w}{n d^2 a}$ the degrees in the arc A D B; for the cosine of the half of which ($= \angle$ A C E,) put s. Then, by the note to Problem 8, Section 2, Part 1, we have $1 : \frac{d}{2} :: s : \frac{s d}{2} =$ E C. Hence D C − E C = E D, the distance required.

EXAMPLE.

Required the distance D E from the extremity of the diameter D F, where the line A B must be drawn at right angles, when the sector A D B C A is worth 74.183544 shillings; the diameter being 20 chains, and the value per acre of the field 8 shillings.

$$\text{First, } \frac{r\,w}{n\,d^2\,a} = \frac{360 \times 74.183544}{8 \times 400 \times .7854} = \frac{26706.075840}{251.32800}$$

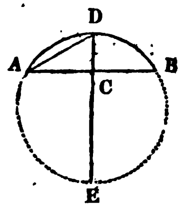
$= 106,26$, the degrees in the arc A D B; and $\frac{106.26}{2} =$ 53,13 is the angle A C E; the cosine of which, to radius 1, is $.6 = s$; whence $\frac{s\,d}{2} = \frac{20 \times .6}{2} = 6. =$ E C, and D C − C E $= 10 - 6 = 4 =$ D E, the distance required.

Note.—400 chains are = 40 acres, which is the reason that $8 \times 400 \times .7854 = 251.32800$.

Note.—In order to avoid swelling the book with unnecessary figures, I have, in the above example, and in the four which follow, given the dimensions and value in chains and shillings, as in the respective problems to which they belong, in Section 2, Part 1; consequently, the answer or result to each problem, will be in chains and decimals, shillings and decimals.

PROBLEM 8.

Let it be required to lay off, from a circular field, by a right line or chord, at right angles to the diameter, a segment worth w *shillings, the diameter being* d *chains, and the value of the field* n *shillings per acre.*



SOLUTION.

Let $x = $ D C, the versed sine; then, by the nature of the circle $\overline{d-x} \times x = dx - x^2 = AC^2$, and $AC = \sqrt{dx - x^2}$, also $\sqrt{dx} = AD$. Then by Rule 2, Problem 12, Section 2, Part 1, $(2\sqrt{dx - x^2} + \frac{4}{3}\sqrt{dx}) \times \frac{4}{10} n x = w$; from which equation, by the well known method of trial and error, the value of x is readily found; which value being set off from D towards E, and the line A B drawn at right angles to D E, it is done.

Or, after x is found, with radius $\sqrt{dx}$, and center D, describe two arcs to cross the circumference in A and B; draw A B through C, and it is done.

Cor.—When the value on one side of the diameter is n, and the other m shillings, the theorem will be $(2\sqrt{dx - x^2} + \frac{4}{3}\sqrt{dx}) \times \frac{n+m}{2} = w$; for the quantity q on each side of the diameter being equal, we have $qn + qm = 2q \times \frac{n+m}{2} = w$; hence $(2\sqrt{dx - x^2} + \frac{4}{3}\sqrt{dx}) \times \frac{4x}{10} \times \frac{n+m}{2} = w$ likewise.

EXAMPLE

Suppose the value per acre of a circular field 12 shillings, required the versed sine or height of a segment, that will be worth 53.617324 shillings, the diameter being 20 chains.

Suppose the two numbers 3 and 5.

Then the first position $= (2\sqrt{20 \times 3 - 3^2} + \frac{4}{3}\sqrt{20 \times 3}) \times \frac{4 \times 3}{10} \times 12 = 35.439408$ the result; and $53.617324 - 35.439408 = 18.177916$ the first error. The second position $(2\sqrt{20 \times 5 - 5^2} + \frac{4}{3}\sqrt{20 \times 5}) \times \frac{4 \times 5}{10} \times 12 = 73.56888$ the result; and $73.56888 - 53.617334 = 19.95164$ the second error.

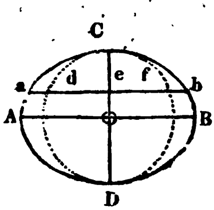
Then $19.95164 \times 3 = 59.85492$
and $18.177916 \times 5 = 90.88958$

their sum $= 150.74450 =$ the products of the errors by the positions. Also $19.95164 + 18.17791 = 38.12955$ the sum of the errors.

Hence, $150.74450 \div 38.12955 = 3.95$, the versed sine, which should have been exactly 4, the deficiency arising chiefly from the waste of decimals by division, and the extraction of roots.

PROBLEM 9.

Let it be required to lay off from an elliptic field, by a double ordinate parallel to the transverse axe, a segment worth w shillings; the transverse and conjugate axes being t and c chains; and the value per acre of the field n shillings.



A a

SOLUTION.

Let A C B D A represent the elliptic field, a C b a the segment; then by Problem 16, Section 2, Part 1, $t : c : : w : \frac{w\,c}{t} = V$ the value of the circular segment d C f d, whose versed sine or height C e is common to both; for which put x, then, by Rule 2, Problem 12, Section 2, Part 1, we have $(2\sqrt{c\,x - x^2} + \frac{4}{3}\sqrt{c\,x}) \times \frac{4\,n\,x}{10} = V$, from whence, by proceeding as in the last example, x is found, and consequently the points d and f, through which the line a b is readily drawn.

Note.—When the value on one side of the conjugate axe is n, and the other m shillings, the area of the segment must be multiplied by $\frac{n+m}{2}$ for the value as mentioned in the last Problem.

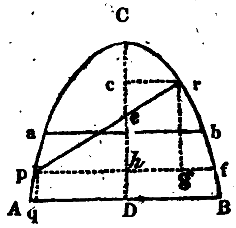
EXAMPLE.

Suppose an elliptic field, whose transverse and conjugate axes are 30 and 20 chains, be worth 12 shillings per acre, what will be the height of a segment cut off by a double ordinate, parallel to the transverse axe, that will be worth 80.425986 shillings.

First 30 : 20 : : 80.425986 : 53.617324 = V, the value of the circular segment d C f d, which value being the same as that in the last example, the versed sine or height will be found the same, by proceeding in the same manner, *viz.* equal to 4 chains.

PROBLEM 10.

Let it be required to lay off, from a field in the form of a parabola, by a double ordinate, perpendicular to its axis, a piece of land next the base, worth w *shillings; the abscissa and ordinate being* a *and* b *chains, and the value per acre of the field* n *shillings.*



SOLUTION.

Let A B C A represent the field, a b the line to be drawn; then, by Problem 17, Section 2, Part 1, $\frac{2}{3} a b =$ the area of the parabola A B C A, and $\frac{2}{3} a b n$ its value. Hence, $\frac{2}{3} a b n - w$ will be the value of the parabola a b C a, for which put V.; then, if x be put equal C e, we have, by the property of the parabola $a : x :: b^2 : \frac{b^2 x}{a}$ the square of the ordinate a b; whence $\sqrt{\frac{x b^2}{a}} \times \frac{2x}{3} \times \frac{n}{10} = \text{V}$, from which equation, by reduction, is found $x = \sqrt[3]{\frac{900 \text{ V}^2 a}{4 n^2 b^2}}$; and $a - x =$ D e the perpendicular breadth to be set off, from D on the abscissa D C.

Note.—When the value on one side of the abscissa is n, and the other m shillings, the area must be multiplied by $\frac{n+m}{2}$ as in the circular and elliptic segments.

EXAMPLE.

Suppose the abscissa and ordinate of a parabolic field be 9 and 5 chains, and the value per acre of the field 20 shillings, what will be the breadth of a piece laid off parallel to the base that is worth 20 shillings.

Here $a = 9$, $b = 5$, $n = 20$, and $w = 20$.

Then $\frac{2}{3}\ a\ b\ n = \frac{2}{3}\ 9 \times 5 \times \frac{1}{10} \times 20 = 60$ the value of the whole field; and $\frac{2}{3} a\, b\, n - w = 60 - 20 = 40$ the value of the remaining part; also $\sqrt[3]{\frac{900\, V^2 a}{4\, n^2\, b^2}} = \sqrt[3]{\frac{900 \times 40 \times 40 \times 9}{4 \times 20 \times 20 \times 5 \times 5}}$ $= \sqrt[3]{324} = 6.869 = x =$ C e the abscissa of the remaining part. Hence, $a - x = 9 - 6.869 = 2.131 =$ D e, the breadth required.

PROBLEM 11.

Let it be required to lay off the same value by the line p r *drawn from the point* p, *oblique to the axis* B C, *at a given perpendicular distance* p q, *from the base or ordinate* A B; *the value, &c. being the same as in the last Problem.*

SOLUTION.

The distance p q = D h (see the Figure to the last Problem) being given, we, of course, have C h given; whence, by conics $a : b^2 : : Ch : \frac{Ch \times b^2}{a} = pf^2$; whence, $pf = \sqrt{\frac{Ch \times b^2}{a}}$, and the area p C f p $= \sqrt{\frac{Ch \times b^2}{a}} \times Ch \times \frac{2}{3}$ for which put A. Hence $A\,n =$ its value; also the value of p C r p = V.—(See the last Problem.) Hence

$A\,n : V :: p f^3 \; (= d^3) :: \frac{V d^3}{n A} = p g^3$, by the property of the parobola;* consequently $p g = \sqrt[3]{\frac{V d^3}{A n}}$; and $p f - p g = p f - \sqrt[3]{\frac{V d^3}{A n}} = g f$; also $\frac{1}{2} p f - g f = h g = c r$, for which put l; then again by conics $h f^2 = (\frac{p f}{2})^2 : l^2 :: C h : C c = \frac{l^2 \times C h}{(\frac{1}{2} p f)^2}$. Hence, $C h - C c = c h = r g =$ the length of the perpendicular to be erected from the point g; lastly draw $p r$, and it is done.

* Demonstration.—Put $x = C c$, $a = C h$, and $b = h f$. Then $b \sqrt{\frac{x}{a}} = c r = h g$, and $b + b \sqrt{\frac{x}{a}} = p g$. Now the fluxion of $c r$ multiplied by $r g \; (a - x) = \frac{b \dot{x}}{2 \sqrt{a x}} \times \overline{a - x}$ is the fluxion of the area $C h g r C$; $\therefore$ that area $= b \sqrt{\frac{x}{a}} \times \overline{a - \frac{1}{3} x}$; and the area $r g f r = \frac{2}{3} a b - b \sqrt{\frac{x}{a}} \times \overline{a - \frac{1}{3} x}$; but the triangle $p g r = \frac{r g}{2} \times p g = \frac{a - x}{2} \times \overline{b + b \sqrt{\frac{a}{x}}}$; $\therefore$ the whole area $p f r p = \frac{2}{3} a b - b \sqrt{\frac{x}{a}} \times \overline{a - \frac{1}{3} x} + \frac{a - x}{2} \times \overline{b + b \sqrt{\frac{x}{a}}}$; and the area $p r C p = \frac{2}{3} a b + b \sqrt{\frac{x}{a}} \times \overline{a - \frac{1}{3} x} - \frac{a - x}{2} \times \overline{b + b \sqrt{\frac{x}{a}}}$; $\therefore$ the area of the whole Parabola $p f C p$ is to the area $p r C p :: \frac{4}{3} a b : \frac{2}{3} a b + b \sqrt{\frac{x}{a}} \times \overline{a - \frac{1}{3} x} - \frac{a - x}{2} \times \overline{b + b \sqrt{\frac{x}{a}}} :: 4 a : 2 a + \overline{3 a - x} \sqrt{\frac{x}{a}} - \frac{3 a - 3 x}{2 \sqrt{a}} \times \overline{\sqrt{a} + \sqrt{x}}$ $:: 8 a \sqrt{a} : 4 a \sqrt{a} + \overline{6 a - 2 x} \sqrt{x} - \overline{3 a - 3 x} \times \overline{\sqrt{a} + \sqrt{x}} :: 8 a \sqrt{a} : 4 a \sqrt{a} + 6 a \sqrt{x} - 2 x \sqrt{x} - 3 a \sqrt{a} - 3 a \sqrt{x} + 3 x \sqrt{a} + 3 x \sqrt{x} :: 8 a \sqrt{a} : a \sqrt{a} + 3 a \sqrt{x} + 3 x \sqrt{a} + x \sqrt{x} :: 8 a \sqrt{a} : (\sqrt{a} + \sqrt{x})^3 :: 8 b^3 : \frac{b^3}{a \sqrt{a}} \times (\sqrt{a} + \sqrt{x})^3 :: (2 b)^3 : (b + b \sqrt{\frac{x}{a}})^3 :: p f^3 : p g^3$.—Q. E. D.

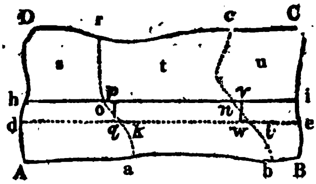
EXAMPLE.

Suppose $p\,q = 1$ chain, every thing else remaining the same as in the last Example.

First $a - p\,q = 9 - 1 = 8 = C\,h$, then $\sqrt{\dfrac{C\,h \times b^2}{a}} = \sqrt{\dfrac{8 \times 5 \times 5}{9}} = \sqrt{22.2222} = 4.714 = p\,f$, and $p\,f \times C\,h \times \dfrac{2}{3} \times \dfrac{1}{10} = 4.714 \times 8 \times \dfrac{2}{30} = 2.5141 = A$. Also, $A\,n = 2.5141 \times 20 = 50.282$ shillings. Now V being equal 40 shillings, (see the last example,) we next have $\sqrt[3]{\dfrac{V\,d^3}{A\,n}}$ (putting $d = p\,f$) $= \sqrt[3]{\dfrac{40 \times 4.714 \times 4.714 \times 4.714}{50.282}} = 4.3679$ nearly $= p\,g$; and $p\,f - p\,g = 4.714 - 4.3679 = .3461 = g\,f$. Also $\dfrac{1}{2}\,p\,f - g\,f = 2.357 - .3461 = 2.0109 = h\,g = c\,r$, (for which put l.) Then $\dfrac{l^2 \times C\,h}{(\frac{1}{2}\,p\,f)^2} = \dfrac{2.0109 \times 2.0109 \times 8}{2.357 \times 2.357} = 5.825 = C\,c$; and $C\,h - C\,c = 8 - 5.825 = 2.175 = r\,g$ the perpendicular to be erected from the point g in the ordinate $p\,f$.

PROBLEM 12.

Let it be required to lay off a piece of land worth any number of shillings, from an irregular field or common of three values, by a right line drawn through them all, so as to make the two ends of the piece cut off, appear of equal breadth, the quality lines being crooked, and oblique one to another.



SOLUTION.

Let A B C D represent the field, a r, b c the lines of partition or quality lines, s, t, and u the value per acre of each flat, and h i the required line. Then, by drawing the guess line d e, and reducing the crooked sides in each piece cut off, to straight ones, by the directions in Section 2, Part 2, we find (by Problem 4, Section 2, Part 1,) the separate area of each of the pieces A a k d, a b l k, and B b e l, and since the value per acre of each piece or part is given, the difference between the sum of the values laid off by guess, and the required value to be laid off, is likewise given, for which put w; then, by drawing the lines q p, and v w, perpendicular to the guess line, so as to reduce the parts to be added or substracted to ▭s of the same area, and determining the distances d q, q w, and w e, for which put d, e, and f, and x equal the breadth wanted, we have, by Problem 2, $x = \frac{w}{ds + et + fu}$ the breadth to be added or substracted parallel to the line d e; consequently, by finding the area and value of each of the last three pieces, and adding or substracting them to or from their respective pieces laid off by guess in each flat, if their sum or difference be equal the required value to be laid off, draw h i, and it is done.

EXAMPLE.

Suppose it was required to lay off, from the field, a piece of land worth 60 shillings, when $s = 12$, $t = 10$, and $u = 6$ shillings per acre.

First, by drawing the guess line d e, and reducing the

crooked sides to straight ones, according to the directions in Section 2, Part 2, I find the area of A h k d, a b l k, and B b e l = 2.32012, 2.04133, and 1.20400, (by Problem 4, Section 2, Part 1,)

and, $\left\{\begin{array}{l} 2.32012 \times 12 = 27.84144 \\ 2.04133 \times 10 = 20.41330 \\ 1.20400 \times \ \ 6 = \ \ 7.22400 \end{array}\right\}$ shillings, their respec-

$$\overline{55.47874}$$

tive values. Consequently $60-55.47874 = 4.52126 (=w)$ shillings the value to be added. Hence, by drawing q p and v w, I find $d = 400$, $e = 410$, and $f = 220$.

Then $\left.\begin{array}{l} ds = 400 \times 12 = 4800 \\ et = 410 \times 10 = 4100 \\ fu = 220 \times \ \ 6 = 1320 \end{array}\right\}$ the length of each piece

into its value; and $\dfrac{w}{ds+et+fu} = \dfrac{452126}{4800+4100+1320} =$

$\dfrac{452126}{10220} = 44.24$ links nearly, the breadth to be added parallel to the guess line.

PROOF.

$\left.\begin{array}{l} 400 \times 44.24 + 2.32012 = 2.49708 \text{ the area of A a o h} \\ 410 \times 44.24 + 2.04133 = 2.22301 \text{ the area of a b p o} \\ 220 \times 44.24 + 1.20400 = 1.30132 \text{ the area of b B i p} \end{array}\right\}$ conse-

quently, $\left\{\begin{array}{l} \text{A.} \qquad\qquad\quad \text{SH.} \\ 2.497 \times 12 = 29.964 \\ 2.223 \times 10 = 22.230 \\ 1.301 \times \ \ 6 = \ \ 7.806 \end{array}\right\}$ their respective values; and

the quantity 6.021, also 60.000 the value laid off, which is equal the required value.

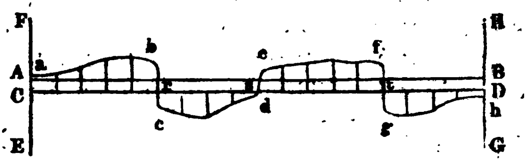
Note.—After h i is drawn, and the value of the three pieces A a o h, a b n o, and b B i n is found and compared with the value to be laid off,—if they are not equal, the breadth of their difference, to be added or substracted, is again approximated by Problem 2, repeating the operation, until the value to be laid off is determined to the greatest exactness.

The above method is that employed by every practical surveyor in dividing

commons, open fields, &c., and is, without doubt, far preferable to that of laying off a piece, first by guess, and then guessing again—no general rule having yet been obtained, nor is it likely any ever will, on account of the irregular bendings, and the obliquity of the partition lines.

PROBLEM 13.

Given the lengths of, and the respective distances between, a certain number of perpendiculars erected on each side of a line that intersects a serpentine fence, and terminates in two other lines or fences perpendicular to it; to determine the position of a right line, parallel to that whereon the perpendiculars are erected, when the sums of the areas of the curved pieces on each side thereon are equal.



SOLUTION.

Let C D represent the line on which the perpendiculars are erected, E F, G H, the fences it terminates in, and a, b, c, d, e, f, g, h, the serpentine fence; then, since the distances between the perpendiculars are given, as well as their lengths, the length of the chord, and also the area of each piece are likewise given. Hence, let a, b, c, d denote the chord, and e, f, g, h the area of each piece; also let $x =$ C A the breadth to be added; then, by the question, it appears, the sum of the areas on one side of the line, is greater than that on the other: consequently, if we take the sum of e and g the greater, we have $e - a x + g - c x = f + b x + h + d x$, from whence, by transposition, we likewise have $e + g - f - h = b x + d x + a x + c x$, and $x = \dfrac{e + g - f - h}{a + b + c + d}$ the breadth to be added or substracted, parallel to the line measured.

Cor.—If the value per acre of each piece, be denoted by k, l, m, n, the theorem will be $\frac{ek+gm-fl-hn}{bl+dn+cm+ak}$.

EXAMPLE.

Suppose the lengths of the perpendiculars, beginning at C in the first part, to be 15, 20, 25, 35, 40, 35; in the second, 25, 35, 40, 15, 11; in the third, 24, 30, 35, 35, 43, and 30; and the fourth, 35, 40, 42, 30, and 21 links, and erected at the distance of 40 links from each other, what is the distance C A or D B, when the sum of the areas on each side of A B are equal?

First $Cr = 40 \times 5 = 200$ the length of the first piece,
$rs = 40 \times 4 = 160$ the length of the second,
$st = 40 \times 5 = 200$ the length of the third,
$tD = 40 \times 4 = 160$ the length of the fourth; and $Cr + rs + st + tD = CD = \underline{\overline{720}}$ the whole length of the line measured.

Then, by Rule 1, Problem 4, Section 3, Part 2, $\frac{\frac{15+35}{2}+20+25+35+40}{5} \times 200 = 5800$, the area of C $a\,b\,r$. Also, $\frac{\frac{25+11}{2}+35+40+15}{4} \times 160 = 4320$, the area of $r\,c\,d\,s$. Likewise, $\frac{\frac{24+30}{2}+30+35+35+43}{5} \times 200 = 6800$, the area of $s\,e\,f\,t$. And, $\frac{\frac{35+21}{2}+40+42+30}{4} \times 160 = 5600$, the area of $t\,g\,h\,t$. Hence, $\frac{e+g-f-h}{a+b+c+d} =$

$\frac{5800 + 6800 - 4320 - 5600}{200 + 160 + 200 + 160} = \frac{2680}{720} = 3.72$ links, the distance required.

Note.—The above Theorem holds good, whether the offsets are, or are not, at equal distances one from another.

EXAMPLE 2.

Required the distance C A or D B, when the value per acre of the first part = 10; of the second 8; of the third 20; and of the fourth 15 shillings; every thing else remaining as before.

$$\begin{array}{rl} \text{First, } ek = 5800 \times 10 = 58000 \\ gm = 6800 \times 20 = 136000 \end{array}\Big\} \; 194000$$

$$\begin{array}{rl} -fl = 4320 \times 8 = 34560 \\ -hn = 5600 \times 15 = 84000 \end{array}\Big\} - 118560$$

$$\begin{array}{rl} \text{And, } bl &= 160 \times 8 = 1280 \\ dn &= 160 \times 15 = 2400 \\ cm &= 200 \times 20 = 4000 \\ ak &= 200 \times 10 = 2000 \\ \hline & \qquad\qquad 9680 \end{array}$$

Then, $\frac{ek + gm - fl - hn}{bl + dn + cm + ak} = \frac{58000 + 136000 - 34560 - 84000}{1210 + 2400 + 4000 + 2000}$ $= \frac{194000 - 118560}{9680} = \frac{75440}{9680} = 7.79$ links, the distance required.

SECTION VIII.

OF FINDING DIMENSIONS NECESSARY IN DESCRIBING A FIGURE OF A GIVEN AREA.

PROBLEM 1.

To find the side of a Square that will contain a given area.

RULE.

Extract the square root of the given area, and it will be the length of the side required. For let $a =$ the given area, and $x =$ the required side; then, by Problem I, Section 2, Part 1, $x^2 = a$, whence $x = \sqrt{a}$ the length of the required side. Q. E. D.

EXAMPLE.

Required the side of a square, that will contain two acres of land,

First, $100000 \times 2 = 200000$ square links, the area.

Then, $\sqrt{200000} = 447.202$, the side required.

PROBLEM 2.

To find the side of a rectangular Parallelogram, that will contain a given area, the ends being given.

RULE.

Divide the area, by the end, and the quotient will be the required side.

RULE 2.

When the side is given, divide the area by the given side, and the quotient will be the end.

EXAMPLE.

Required the length of a side of a rectangular parallelogram, that will contain one acre of land, the ends being each 200 links.

By Rule 1, $\frac{100000}{200} = 500$ links, the length of the side required.

By Rule 2, when the side is 500, and the area 100000, we have $\frac{100000}{500} = 200$ links, the breadth of the end.

DEMONSTRATION.

Let $a =$ the given area, $b =$ the length of one of the ends, and $x =$ the required side; then, by Problem 1, Section 2, Part 1, $b\,x = a$; whence $x = \frac{a}{b}$ the length of the side required.

Cor.—When x is given, and b required, we have, by the same Problem, $b = \frac{a}{x}$ for the length of the end.

PROBLEM 3.

To find the base and perpendicular of a right-angled Triangle, that will contain a given area, when they are equal to each other.

RULE.

Extract the square root of double the given area, and the root will be the length of each.

EXAMPLE.

Required the base and perpendicular of a right angled triangle, (when equal to each other,) that will contain one acre of land.

First, $100000 \times 2 = 200000$ square links, the double area.

Then, $\sqrt{200000} = 447.202$ links, the length of each.

RULE 2.

When the base or perpendicular is given, divide double the given area, by the given part, and the quotient will be the required part.

EXAMPLE.

Required the perpendicular of a triangle, that will contain two acres of land, the base being 400 links.

First, $200000 \times 2 = 400000$ links, the double area; then, $\frac{400000}{400} = 1000$ links, the perpendicular required.

DEMONSTRATION.

Let $a =$ the given area, and $x =$ the base, also equal the perpendicular. Then, by Problem 2, Section 2, Part 1, $\frac{x x}{2} = a$; whence, $x = \sqrt{2 a}$, the length of each required.

Cor.—When either the base or perpendicular is given, we have, (by putting the given part $=$ to b,) $x = \frac{2 a}{b}$ the required part; and the same holds true in all plain triangles.

PROBLEM 4.

To find the side of an equilateral Triangle, that will contain a given area.

RULE.

Divide 16 times the square of the given area by 3; ex-

tract the biquadrate root of the quotient, and it will give the length of the side required.

EXAMPLE.

Required the side of an equilateral triangle, that will contain 3 acres of land.

First, $100000 \times 3 = 300000$ square links, the area; and $300000 \times 300000 = 90000000000$ the square of the area. Hence, $\sqrt[4]{\dfrac{90000000000 \times 16}{3}} = \sqrt[4]{\dfrac{1440000000000}{3}} = \sqrt[4]{480000000000} = 832.35$ links, the length of the side required.

DEMONSTRATION.

Let $a =$ the given area, and $x =$ the required side, then (by Euclid 47.1,) we have the perpendicular height of the triangle $= \sqrt{x^2 - \dfrac{x^2}{4}} =$ (by Problem 2,) $\dfrac{2a}{x}$; consequently $x^2 - \dfrac{x^2}{4} = \dfrac{4a^2}{x^2}$; and by reduction, $x = \sqrt[4]{\dfrac{16a^2}{3}}$ the length of the side.—Otherwise thus.

Let $x =$ the side, then (by Cor. 2, Rule 3, page 98, Hutton's Mensuration,) we have $\sqrt{\dfrac{3x^4}{16}} = a$. Hence $\dfrac{3x^4}{16} = a^2$, and $x = \sqrt[4]{\dfrac{16a^2}{3}}$, the length of the side as before.

PROBLEM 5.

To find the sides of a Trapezium, when in Arithmetical Progression, that will contain a given Area.

RULE.

First suppose any four numbers in arithmetical progression, (as 1, 2, 3, and 4,) to represent the sides of a similar trapezium, and find its area by the Note to Problem 4, Section 2, Part 1. Then the quotient of the square root of the given area, divided by the square root of the supposed area, when multiplied by each respective number in the progression, will give the length of each side.

EXAMPLE.

Required the lengths of the sides of a trapezium, when in arithmetical progression, that will contain 3 acres, 2 roods, and 16 perches of land.

First $\sqrt{100 \times 200 \times 300 \times 400} = \sqrt{2400000000} =$ 48989.8 square links, the supposed area; and 3 acres, 2 roods, 16 perches $= 360000$ square links, the given area; then $\frac{\sqrt{360000}}{\sqrt{48989.8}} \times 100, 200, 300,$ and $400 = 2.71 \times 100,$ 200, 300, and 400 respectively $= 271, 542, 813,$ and 1084 links, the sides required.

DEMONSTRATION.

Let x, y, z, and w, represent the required sides, and a equal the given area; also let 1, 2, 3, and 4 represent the sides of a similar trapezium, and $b =$ its area. Then, since similar figures are as the square of their like sides, or the sides as the square roots of their areas, we have $\sqrt{b}$:

$\sqrt{a} : : 1 : x : : 2 : y : : 3 : z : : 4 : w$. Hence $x = \frac{\sqrt{a}}{\sqrt{b}}$, $y = \frac{2\sqrt{a}}{\sqrt{b}}$, $z = \frac{3\sqrt{a}}{\sqrt{b}}$, and $w = \frac{4\sqrt{a}}{\sqrt{b}}$, the length of each side.—Q. E. D.*

Note.—When one of the sides is given, to lay out upon it a given area, we have this

RULE OR CONSTRUCTION.

On the given side, as a base of a right-angled triangle, find (by Cor. Problem 3,) the perpendicular to it, when the said triangle contains any part of the given area; after that, take the perpendicular thus found for a base, and find the perpendicular belonging to it, by the above Cor., that will constitute another triangle, containing the remaining part of the area: draw lines from one extremity to another around the figure, and it is done.

Or, after finding the first perpendicular, (by Euclid 47.1,) find the hypothenuse, and on it as a base, constitute the other triangle, then draw lines, &c.

Likewise, when the diagonal is given, we have this

RULE.

From the given area, and diagonal as a base, find the perpendiculars by the above Cor. quoted, and erect them on each side of the diagonal; afterwards draw lines from one extremity to another, and it is done.

PROBLEM 6.

To find the diameter of a Circle, that will contain a given Area.

RULE.

Divide the given area by .7854, extract the square root of the quotient, and the root will be the diameter required.

* For a general method of construction, see Simpson's Algebra, Problem 81, page 398.

EXAMPLE.

Required the diameter of a circle that will contain half an acre of land.

First $\frac{50000}{.7854} = 63662$, the square of the diameter, and $\sqrt{63662} = 252.3$ links, the diameter required.

DEMONSTRATION.

Let $a =$ the given area, $b = .7854$, and $x =$ the diameter; then (by Rule 2, Problem 9, Section 2, Part 1,) $x^2 b = a$; whence $x = \sqrt{\frac{a}{b}}$ the diameter required.

PROBLEM 7.

To find the side of any regular Polygon, that will contain a given Area.

RULE.

Divide four times the given area, by the tangent of half the angle of the polygon multiplied by the number of sides; the square root of the quotient will be the length of the side required.

EXAMPLE.

Required the side of a hexagon, that will contain an acre of land.

First $\overline{6 - 2} \times \frac{90}{6} = \frac{4 \times 90}{6} = \frac{360}{6} = 60° = \frac{1}{2}$ the angle of the polygon; the tangent of which (by the Table to

Problem 6, Part 2,) = 1.7320508. Hence, $\sqrt{\dfrac{100000 \times 4}{1.7320508 \times 6}}$ $= \sqrt{38490.03} = 196.188$ links, the length required.

DEMONSTRATION.

Let $t =$ the tangent of half the angle of the polygon to radius 1, $n =$ the number of sides, $p =$ the perpendicular, $a =$ the given area, and $x =$ the length of a side. Then, by Trig. Rad. $1 : t :: \frac{x}{2} : p = \frac{t\,x}{2}$, consequently $\frac{t\,x^2}{4}$ = the area of a triangle, whose base is the side x, and $\frac{t\,x^2}{4} \times n = a$ the area of the whole polygon; from whence is had $x = \sqrt{\frac{4\,a}{t\,n}}$ the length of a side.—Q. E. D.

Cor.—Half the angle of the polygon is equal $\overline{n-2} \times \frac{90}{n}$. For the angle at the center $= \frac{360}{n}$ half of which is $\frac{360}{2\,n}$; consequently the complement of the said half is $\frac{360}{4} - \frac{360}{2\,n} =$ by reduction, $\overline{n - 2} \times \frac{90^\circ}{n}$.

PROBLEM 8.

To find the distance between the circumferences of two concentric Circles, when the Area included between them and the Area of the lesser Circle are given.

RULE.

Divide the given area by 3.1416; to the quotient add $\frac{1}{4}$ the square of the lesser diameter; from the square root of their sum substract $\frac{1}{2}$ the lesser diameter; the difference will be the distance required.

RULE 2.

Find the diameters of both circles by Problem 6, then half their difference will be the distance required.

EXAMPLE.

Required the distance when the space included between the circumferences will just contain two acres of land, the area of the lesser circle being half an acre.

First by Problem 6 $\sqrt{\frac{50000}{.7854}} = \sqrt{63662} = 252.3$ the diameter of the lesser circle; then by Rule 1, $\sqrt{\frac{200000}{3.1416} + \frac{63662}{4}} - \frac{252.3}{2} = \sqrt{63662 + 15915.5} - 126.15 = \sqrt{79577.5} - 126.15 = 282.09 - 126.15 =$ 155.94, the distance required.

DEMONSTRATION.

Let $a =$ the given area, $b = .7854$, $c =$ the area of the lesser circle, and $x =$ the required distance; then, since the area of the lesser circle is given, its diameter is likewise given by Problem 6, for which put d; then will $d + 2x$ be the diameter of the greater, and (by Problem 10, Section 2, Part 1,) $\overline{2d + 2x} \times 2xb = 4bdx + 4bx^2 = a$; whence $x^2 + dx = \frac{a}{4b}$ and, by completing the square, $x = \sqrt{\frac{a}{4b} + \frac{d^2}{4}} - \frac{d}{2}$, the distance required.

PROBLEM 9. (CARNAN'S DIARY.)

To find the two Radii, that will describe the Lune of Hippocrates, so as to contain a given Area.

RULE.

Extract the square root of the given area, likewise that of twice the given area; the two roots of which will be the radii required.

EXAMPLE.

Required the two radii, that will describe the Lune of Hippocrates, so as to contain 1 A. 2 R. 16 P. of Land.

First 1 A. 2 R. 16 P. $=160000$ square links; hence the $\sqrt{160000}=400$ links the lesser radii; and $\sqrt{2\times 160000}$ $=\sqrt{320000}=565.68$ the greater.

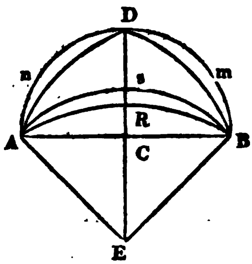
DEMONSTRATION.

Let $a=$ the given area, $x=$ the lesser radius, (or radius of the outer circle,) and $y=$ the radius of the greater. Then (by the Art. Lune, Page 58, Hutton's Dict. Vol. 2.) we have $x^2=a$; and, by Euclid 47.1, $\sqrt{2xx}=y$; whence $x=\sqrt{a}$ and $y=\sqrt{2a}$ the lengths of the radii required.

Cor.—When $a=1$, then $x=1$, and $y=\sqrt{2}=1.41412$.

Cor. 2.—It appears from the above proportion the two radii bear to each other, that if semiparabolic arcs (having their respective abscissa and ordinate equal the above radii,) be described, instead of circular arcs, the area so circumscribed by the parabolic arcs, will be equal the

area circumscribed by the circular arcs. For, by the property of the lune in the book quoted above, A C = C B = C D = C E, for which put x; also let y = A E. Then, by Problem 17, Section 2, Part 1, $\frac{2x^2}{3} + \frac{2x^2}{3} = \frac{4x^2}{3}$ the area of the two semiparabolas A D C A and B D C B. And since, by Euclid 47.1, $y = \sqrt{2x^2}$, we have $\frac{2}{3} \times 2x^2 = \frac{4x^2}{3}$ the area of the semiparabola A R B E A, consequently, the areas being equal, and the segment A R B C A common to both, it is evident, if the said segment be substracted, that the figure A D B R A will be equal the triangle A B E, which triangle is equal the lune A n D m B s A, or the area to be laid off.



PROBLEM 10.

To find the two Axes of an Ellipsis, that will contain a given Area, when they are in any given ratio one to another.

RULE.

Multiply the given area, by the antecedent of the given ratio; divide the product, by the consequent multiplied by .7854, and the square root of the quotient will be one axis.

The other is found likewise, by multiplying the area by the consequent, and dividing by the antecedent, multiplied by .7854, &c.

EXAMPLE.

Required the two axes of an ellipsis that will contain one acre of land, when the transverse is to the conjugate as 3 to 2.

First $\sqrt{\dfrac{100000 \times 3}{.7854 \times 2}} = \sqrt{\dfrac{300000}{1.5708}} = \sqrt{190985.48}$ $= 437.018$ the transverse axis; and $\sqrt{\dfrac{100000 \times 2}{.7854 \times 3}} =$ $\sqrt{\dfrac{200000}{2.3562}} = \sqrt{84882} = 291,345$ the conjugate.

DEMONSTRATION.

Let $a =$ the given area, $b = .7854$, $x =$ the transverse axis, $y =$ the conjugate, and m and n the ratio they have to each other. Then, by Problem 15, Section 2, Part 1, $m\,n\,b$ will be the area of a similar ellipse; and by Cor. 9, Problem 5, Page 291, Hutton's Mensuration, $m\,n\,b : a :: m^2 : x^2 = \dfrac{a\,m}{b\,n}$ and $m\,n\,b : a :: n^2 : y^2 = \dfrac{a\,n}{b\,m}$. Hence, $x = \sqrt{\dfrac{a\,m}{b\,n}}$ and $y = \sqrt{\dfrac{a\,n}{b\,m}}$ the axes required.—Q.E.D.

Cor.—If either of the axes are given to find the other, when the ellipsis is to contain a given area, we have, by putting $a =$ the area, $b = .7854$, $c =$ the given axis, and $x =$ the required one, $c\,b\,x = a$, or $x = \dfrac{a}{c\,b}$.

SECTION IX.

OF DIVIDING A COMMON, COMPRISED OF DIFFERENT VALUES, AMONG A NUMBER OF PROPRIETORS, ACCORDING TO THEIR RESPECTIVE CLAIMS, WITH THE METHOD OF KEEPING THE BOOKS FOR THE SAME.

DIRECTIONS.

Survey and plan the common to be divided, according to the directions laid down, (Section 4,) due regard being had to all highways, foot paths, and hedges passed; all other particular objects being likewise laid truly upon the plan; and a greater number of ⊙s. being made than at other times, in order to determine the true position of the quality lines, as well as assist in staking out the allotments. Then mark off the highways, quarries, or gravel pits, and lay down the quality lines* upon the plan, where the commissioners have fixed them upon the ground; which lines must be dotted ones, dividing the common into a number of parts called flats. Next number the flats, and find the area† and value of each, clear of roads, &c. the

* It is to be observed, in staking out the quality lines, that double stakes must always be put down at the crossing or intersection of the lines, and a long narrow sod cut out of the ground from the foot of them, to denote the direction of each line, and, in order to get them properly transferred to the plan, they are traced the same as fences in old Inclosures, by measuring one ⊙ or △ to another,—taking the offsets, &c. as before directed.

† As too much accuracy cannot be used both before and in alloting, I make it a general rule to cast all the flats twice over, lest I should commit an error; after that, I divide the common or field, to be allotted into a number of trapeziums and triangles, and so find the area again for a further correction.

same as a single field, and enter them into the flat book; then collect the flats, and find the gross quantity and value to be divided.

The claims being settled, next begin your deductions, and, by the Rule of Fellowship, find each proprietor's claim upon the common, for land and houses. After completing your deductions, and finding the net value of each proprietor's claim, (by Problem 12, Section 7, Part 2,) allot to each proprietor his respective share of the common upon the plan, and into each allotment put the owner's name, its tenure, and the quantity it contains in acres, roods, and perches; also mark what part of the division fences the said owner is to make, and afterwards keep in good repair.

BOOKS USED IN ALLOTING.

When a common or piece of ground, comprised of different values, is to be divided among the proprietors of land and houses, in the township it belongs, it is necessary to have the three following books.

BOOK 1.—THE PROPERTY BOOK.

This book is the same in form as the survey book given in Section 6; and to avoid introducing unnecessary matter, I have considered every occupier in it, a proprietor, and consequently have transferred from thence the value of each proprietor's property, in shillings and decimals, to the deduction sheet.

BOOK 2.—THE FLAT BOOK;

AND

BOOK 3.—THE ALLOTMENT BOOK.

For their use, explanation, and description see further on.

Note.—In dividing an open township field, it is to be observed, that the property book is then made from the flat book, *i. e.* the flat book is first made (after the field is measured, valued, and cast in flats,) containing the number, quantity, value per acre, and total value of each flat. From this the number of ridges, quantity, and value belonging to each proprietor, in each flat, is transferred to the property book, to make up his or her respective claim. From thence it is carried to the deduction sheet, and reduced according to the directions of the act or agreement for dividing the same. The field is then divided in the same manner as a common.

Also, when a township, which is concerned with the division of a common, contains part open field land, it is then partly made from the flat book.

Having given proper directions how and in what manner to proceed in surveying the common, &c. I shall next shew how and in what manner it is to be divided, by the following Act:

AN ACT

For Inclosing and exonerating from Tithes Lands in the Township and Parish of B—, in the —— Riding of the County of York.

[Royal Assent, — *June*, 18—.]

Whereas there are within the manor and township of B—, in the parish of B—, in the — Riding of the county of York, certain common and waste grounds, commonly called or known by the name of B—— common, containing one hundred and fifty-five acres, or thereabouts: Preamble.

And whereas John Blakelock, Esq. is lord of the manor of B— aforesaid, and is also impropriator of the rectory of B—, and the Rev. James Shylock is rector thereof, and as such entitled to the tithes within the said manor:

And whereas the said John Blakelock, Thomas Horn, and others, are owners and proprietors of all the messuages, cottages, frontsteads, lands, grounds, and hereditaments, in the said township of B—, part of which are freehold, and part copyhold:

And whereas an act passed in the forty-first year of the reign of his Majesty King George the Third, intituled, "An act "for consolidating in one act certain provisions usually inserted "in acts of inclosure, and for facilitating the mode of proving "the several facts, usually required on the passing of such "acts:"

And whereas it would be of great advantage to all persons interested in the premises, and of utility to the public, if the said common and waste grounds were inclosed, and exonerated from tythes; but as the same cannot be effected without the aid and authority of parliament,

May it therefore please your Majesty,

That it may be enacted; And be it enacted by the King's most Excellent Majesty, by and with the advice and consent of the Lords spiritual and temporal, and Commons, in this present

Parliament assembled, and by the authority of the same, That the said commons and waste grounds shall be divided, allotted and inclosed in manner hereinafter mentioned; and that T. H. of N—, in the county of York, gentleman, and R. B. of W—, in the said county, gentleman, shall be and they are hereby appointed commissioners for dividing, allotting, inclosing, and otherwise improving the said common and waste grounds, and for carrying this act into execution, with, under, and subject to such of the powers, authorities, regulations, restrictions and provisions of the said recited act as are not repugnant to or otherwise provided for by this act.

Commissioners.

Appointment of new commissioners.

And be it further enacted, That if the said T. H. or any commissioner to be appointed in his stead, as hereafter mentioned, by the lord of the said manor for the time being, shall die, refuse, or become incapacitated, or neglect for the space of six calendar months to act, then, and in every such case, the lord of the said manor, for the time being, shall and may thereupon appoint some other person, not interested in the said inclosure, to be a commissioner in the stead of the said T. H. or of such commissioner so dying, refusing, or becoming incapacitated, or neglecting to act; and if the said R. B. or any commissioner to be appointed by the majority in value of the persons interested in the said lands, present at a meeting as hereinafter mentioned, shall die, refuse, or become incapacitated, or neglect for the space of six calendar months to act, then some other person, not interested in the said inclosure, shall and may be thereupon appointed in the stead of the said R. B. or of any such commissioner dying, refusing, or becoming incapacitated, or neglecting to act as last aforesaid, by a majority in value, (to be ascertained by the land tax assessment,) of the persons interested in the lands hereby intended to be divided and inclosed, who shall be present at a meeting to be holden for that purpose, pursuant to notice to be affixed upon the principal outer door of the church of B— aforesaid, specifying the time and place of such meeting, upon a Sunday, at least ten days before such meeting; and in default of any such appointment of a new commissioner for the space of twenty-eight days after any such death, refusal, or incapacity to act, shall happen or be known, a new commissioner, not interested in the said inclosure, shall be forthwith appointed by the surviving or remaining commissioner; and every such appointment of a new commissioner, shall be reduced into writing, and

signed by the person or persons making the same, and shall be deposited and kept with the award of the said commissioners hereinafter directed to be made; and every commissioner so to be appointed, shall have the like powers and authorities for putting this act, and the said recited act into execution, in all respects as the commissioner, in whose place he shall have been so appointed, was invested with, or as if he had been nominated in and by this act.

And be it further enacted, That the said commissioners shall (after having taken and subscribed the oath or affirmation prescribed by the said recited act, and before they proceed to any other business touching the execution of this act,) and they are hereby authorized and required, to appoint some other person (not interested in the said common, or the inclosure thereof,) to be an umpire for the purposes of this act; and in case such person, or any other person to be appointed as umpire for the purposes of this act, shall die, or refuse or neglect for the space of six calendar months to act, or shall become incapable of acting as such umpire, the said commissioners shall, and they are hereby required forthwith to appoint another person (not interested in the said division and inclosure,) to be an umpire for the purposes of this act; and that whenever the said commissioners shall differ, or disagree in opinion, touching any matter or thing to be by them done or performed, in pursuance of this act, or the said recited act, then, and in every such case, the matter or thing in question shall be determined by the umpire, so to be appointed for the purposes of this act; whose determination therein and thereupon shall be deemed to be the determination of the said commissioners, and shall be acted upon accordingly. Umpire to be appointed.

Provided always, and be it enacted, That no person shall be capable of acting in the execution of this act as umpire, until he shall have taken and subscribed an oath in the form or to the effect following, (that is to say,) Umpire to be sworn.

"I, *A. B.* do swear, [*or*, being one of the people called "Quakers, do solemnly affirm] That I will faithfully, impartially and honestly, according to the best of my skill "and ability, execute and perform the trust, power, and "authority reposed in me as umpire, by virtue of an act, "made in the fifty-seventh year of the reign of King "*George* the Third, intituled, [*here insert the title of this* "*act*,] according to equity and good conscience, and with-

"out favour or affection, prejudice or malice to any per-
"son or persons whomsoever.

"So help me God."

Which oath or affirmation it shall and may be lawful to and for either of the said commissioners to administer, (who is hereby required to administer the same,) and such oath or affirmation when so taken and subscribed by such umpire, shall be annexed to and enrolled with the award of the said commissioners, and a copy of the enrolment thereof, shall be admitted as legal evidence.

Commissioners to give notice of their meetings.

And be it further enacted, That the said commissioners shall and they are hereby required to cause notice to be affixed on the door of the parish church of B— aforesaid, upon some Sunday, before divine service, and also to be published once at least in some newspaper published in the said county of York, of the time and place of their first meeting for executing the powers hereby and by the said recited act vested in them, at least six days before such meeting, and shall in like manner give six days notice of every subsequent meeting, (meetings by adjournment only excepted;) and if at any time appointed for any such meeting, or to which any such meeting shall be adjourned, only one of the said commissioners shall attend, then such commissioner shall and may adjourn the said meeting to such future day as he shall think fit; and all the meetings of the said commissioners shall be held in the said township of B—, or within eight miles thereof, at the discretion of the said commissioners; and the said commissioners and umpire, and also the said proprietors, their agents and solicitors, shall at all meetings for executing this act, pay their own expenses.

Other notices.

And be it further enacted, That all other notices necessary or requisite to be made or given by the said commissioners shall be made and given by advertisement in the newspaper called the T—, or in some other newspaper printed or circulated in the said county of York.

Proceedings to be entered in a book.

And be it further enacted, That all orders and proceedings of the said commissioners shall be entered into a book or books to be provided for that purpose, and shall be signed by such commissioners; and such book and the entries therein of such proceedings respectively, shall and may be read in evidence in all actions at law, concerning any thing done in relation to or in execution of this act.

Commis-

And be it further enacted, That the said commissioners shall

and they are hereby authorized and required, as soon as conveniently may be, to make a true, just and perfect valuation of all the lands and grounds in the said township of B—, or such of them as the said commissioners shall deem necessary to be valued for the purposes of this act; and the valuation so to be made shall be reduced into writing, and signed by the said commissioners making the same, and after the signing thereof, shall, for the purposes of this and the said recited act, be deemed and taken to be the true valuation of all the same lands and grounds respectively. sioners to make a valuation of all the lands in the township.

And be it further enacted, That if any dispute or difference shall arise between any of the proprietors of or persons interested or claiming to be interested in the said intended division and inclosure, touching or concerning the respective claims, estates, rights, or interests, which they or any of them have or claim to have in or to the common and waste grounds hereby directed to be divided and inclosed, or touching or concerning the respective shares or allotments which they or any of them ought to have of or in the same, or touching or concerning any other matter or thing relating to the said division and allotments, it shall be lawful for the said commissioners, and also for the said umpire, in the cases hereinbefore provided, and they and he are and is hereby authorized and required to examine into, hear and determine the same: Provided always, that nothing herein contained shall extend or be construed to extend to enable the said commissioners or umpire to determine the title to any lands, tenements, or hereditaments whatsoever. Commissioners to determine claims.

Provided always, and be it further enacted, That nothing in this act contained shall authorize the said commissioners to determine any right between any parties contrary to the possession of such parties; but in case the said commissioners shall be of opinion against the right of the person or persons so in possession, they shall forbear to make any determination thereon, until the possession shall be given up by or recovered from such person or persons so in possession, by ejectment, or other due course of law. Commissioners not to determine rights contrary to the possession.

And be it further enacted, That in case the said commissioners, or the said umpire, shall upon the hearing and determination of any claim or claims, objection or objections, to be delivered to the said commissioners in pursuance of the said recited act or this act, see cause to award any costs, it shall and Power to assess costs.

may be lawful to and for the said commissioners, or the said umpire, and they respectively are hereby empowered, upon application made to them for that purpose, to settle, assess, and award such costs and charges as they respectively shall think reasonable to be paid to the party or parties in whose favour any determination of the said commissioners, or the said umpire, shall be made, by the person or persons whose claim or claims, objection or objections, shall be thereby disallowed or over-ruled, or against whom the said commissioners or the said umpire shall have determined as aforesaid; and in case the person or persons who shall be liable to pay such costs and charges shall neglect or refuse to pay the same on demand, it shall and may be lawful to and for the said commissioners, and the said umpire respectively, and they are hereby respectively authorized and required, by warrant under their hands and seals directed to any person whomsoever, to cause such costs and charges to be levied by distress and sale of the goods and chattels of the person or persons so neglecting or refusing to pay the same, rendering the overplus (if any) upon demand to the person or persons whose goods and chattels shall have been so distrained and sold, after deducting the costs and charges attending such distress and sale.

Power to try rights by an issue at law.

Provided always, and be it further enacted, That in case any person or persons shall be dissatisfied with any determination of the said commissioners, or the said umpire, touching or concerning any claim or claims of rights, interest, or exemptions, in, over and upon any of the lands and grounds within the said township of B—, it shall and may be lawful to and for such person or persons who shall be so dissatisfied with the determination of the said commissioners or umpire, to cause an action to be brought in one of His Majesty's courts of record at W——, against the person or persons in whose favour such determination shall have been made, within two calendar months, after such determination of the said commissioners or umpire shall have been made and notified in writing to him her or them, and to proceed to a trial at law of the matter so determined by the said commissioners or umpire, upon a feigned issue or issues, at the first assizes to be holden for the said county of York, after the expiration of three calendar months from the time such determination shall have been made and notified as aforesaid; and the defendant or defendants in such action shall and he she or they is and are hereby required

to name an attorney or attornies, who shall appear thereto, or file common bail, and accept one or more issue or issues, whereby the claim or claims, and the right or rights thereby insisted upon, may be tried and determined, such issue or issues to be settled by the proper officer of the court in which such action or actions shall be commenced, in case the parties shall differ about the same, and the verdict or verdicts which shall be given in such action or actions shall be binding, final and conclusive upon all and every person or persons whomsoever, unless the court wherein such action or actions shall be brought shall set aside such verdict or verdicts, and order a new trial to be had therein, which it shall be lawful for the said court to do, in case the said court shall think proper; and after such verdict or verdicts shall be obtained, and not set aside by the said court, the said commissioners shall and they are hereby required to act in conformity thereto, and to allow or disallow the claim or claims thereby determined, according to the event of such trial or trials.

Provided always, and be it enacted, That if any of the parties in any action or actions to be brought and prosecuted in pursuance of this act shall die pending the same, such action or actions shall not abate by reason thereof, but shall be proceeded in as if no such event had happened; and if any person or persons in whose favour any such determination of the said commissioners shall have been made, and against whom any such action or actions might have been brought if living, shall die before any action or actions shall have been brought, and before the expiration of the time hereinbefore limited for bringing such action or actions, it shall be lawful for the person or persons who might have brought such action against the person or persons so dying, to bring the same, within the time so limited as aforesaid, against such person or persons as if actually living, and to serve the clerk of the said commissioners with notice of such action or actions, in the same manner as the party or parties so dying might have been served therewith if living, and it shall then be incumbent on the heir or heirs, or other person or persons who shall claim the benefit of such determination as aforesaid, to appear and defend such action or actions in the name or names of the person or persons so dead, and proceedings shall be had therein in the same manner as if such person or persons had been actually living, and the

Actions not to abate by the death of a party.

How to be brought in case of a previous death.

rights of all parties shall be equally bound and concluded by the event of such action or actions.

Surveyor appointed.

And be it further enacted, That W. R. of R—, in the county of York, land surveyor, shall be and he is hereby appointed surveyor of the common, and also of such of the ancient inclosed lands within the said township and manor of B— as the said commissioners shall think necessary to be surveyed for the purpose of this act; and in case of the death, neglect or refusal to act, or resignation of the said W. R. or of any further surveyor, it shall be lawful for the said commissioners to appoint some other fit and competent person or persons not interested in the said lands and grounds to be the surveyor or surveyors for the purpose of this act; provided always that the said W. R. or such other person or persons so to be appointed surveyor or surveyors aforesaid, before he or they shall proceed to act as such surveyor or surveyors, shall take and subscribe an oath in the form or to the effect following: (that is to say,)

"I do swear (or, being one of the people called "Quakers, do solemnly affirm,) that I will faithfully, im-"partially, and honestly do and execute the several duties "incumbent on me as surveyor, under and by virtue of an "act passed in the —— year of the reign of King "George the Fourth, intituled, 'An act for inclosing lands "within the manor of B—, in the county of York,' ac-"cording to the best of my skill and judgment, and "according to equity and good conscience, and without "partiality, favour or affection, prejudice or malice to any "person or persons whomsoever.

"So help me God."

Which oath or affirmation the said commissioners, or either of them, or any justice of the peace acting for the said riding, are and is hereby empowered and required to administer; and the said oath or affirmation, so taken and subscribed by such surveyor or surveyors, shall be annexed to and enrolled and deposited with the award of the said commissioners; and an office copy of the enrolment thereof, shall be admitted as legal evidence.

Turf or soil not to be cut or taken away without leave of

And be it further enacted, That if any person shall after the passing of this act, cut, dig, pare, grave or take away peat, turf, sods, or soil in, upon or from the said common or waste grounds, or any part thereof, without or contrary to the

license of the said commissioners first had and obtained in writing for that purpose, then and in every such case the said commissioners, upon due proof made to them upon oath (which oath either of the said commissioners is authorized to administer,) shall and they are hereby required, by warrant under their hands and seals, to cause any sum or sums of money, not exceeding five pounds, to be levied by distress and sale of the goods and chattels of the persons so offending in the premises, rendering the overplus (if any) upon demand to the person or persons whose goods and chattels shall have been so distrained and sold, after deducting the costs and charges attending such distress and sale; and the money so to be levied as aforesaid shall be applied towards the costs and charges of obtaining and executing this act. the commissioners.

Plan of allotments, and objections to be heard.

And be it further enacted, That when and as soon as the said commissioners shall have ascertained the respective shares, rights and interests of the several owners and proprietors in the common and waste grounds to be divided and allotted by virtue of this act, and shall have designed and marked out a map or plan of the shares and proportions by them the said commissioners intended to be set out and allotted unto such several owners and proprietors, the said commissioners shall and they are hereby required to cause notice in writing to be given of a time and place of meeting, by affixing such notice on the door of the parish church of B— aforesaid, eight days at the least before such meeting shall be holden, when and where such several owners and proprietors may inspect the said map or plan; and if upon such inspection any of the said owners and proprietors shall be dissatisfied with his her or their allotment or allotments, then and in such case the said commissioners shall at such meeting give notice of one other meeting at the least, to be holden by them for receiving complaints and objections of and to such several allotments, and after hearing and examining into such complaints and objections the said commissioners may either confirm or alter any such allotment or allotments, or any part or parts thereof, and the decision and determination of the said commissioners therein shall be final and conclusive.

Openings to be left.

And be it further enacted, That the said commissioners shall, either before or at the time of making their award, direct and appoint what gaps or openings shall be left in any of the hedges or fences hereby or by the said recited act directed to

be made, and for any and what time, for the passage of cattle, carts and carriages.

Commissioners to regulate water-courses.

And be it further enacted, That the said commissioners shall and may scour out, deepen and widen all such brooks, streams, ditches, water-courses, tunnels and bridges, in the said township of B—, and also shall and may set out, order and direct such new ditches, drains, water-courses, tunnels, water-gates, banks and bridges, to be made of such depth and breadth, and in such situation and direction, as they the said commissioners shall think proper, in, through, over and upon the common and waste grounds hereby intended to be divided and allotted as aforesaid; and the said commissioners shall and may and they are hereby directed in and by their award to order and direct by whom, at whose expense, at what time, and in what manner the said brooks, ditches, drains, water-courses, tunnels, water-gates, banks and bridges shall be thereafter repaired, cleansed, scoured and maintained: Provided always, that no such brook or stream shall be diverted or turned without the consent of the owner or owners, proprietor or proprietors of the lands and grounds from which any such brook or stream shall be diverted, or into which the same shall be turned respectively.

Allotments for stone, &c,

And be it further enacted, that the said commissioners shall and they are hereby authorized and required to set out and allot such part or parts of the said common and waste grounds hereby directed to be divided and allotted, as they shall judge proper, for common ponds, common pits for getting marle, and stone for lime, and for such other purposes as they shall direct and appoint, for the use and benefit of the several proprietors of the allotments to be made of the same common and waste grounds by virtue of this act, under such regulations as the said commissioners shall in and by their award direct and appoint, together with convenient roads to and from the same; and the grass and herbage thereof shall be allotted to such person or persons as the said commissioners shall in and by their said award direct and appoint.

Tythe allotment to the rector.

And be it further enacted, That after setting out and allotting such roads, ways, and other requisites as hereby or by the said recited act are authorized to be set out and allotted in and upon the lands and grounds hereby directed to be divided and inclosed, or any part or parts thereof, the said commissioners shall set out and allot, unto and for the rector of the rectory

of B— aforesaid, such parcel or parcels of land within the same common and waste grounds as shall in the judgment of the said commissioners be equal in value to one sixth part of the then residue of the said common or waste grounds, which allotment shall be and is hereby declared to be a full compensation and satisfaction for all and all manner of tythes, both great and small, and all rectorial, vicarial and ecclesiastical rights, dues, and payments whatsoever, (except as hereinafter mentioned) belonging to him the said rector respectively, arising, growing, renewing, increasing, payable or happening within, for, or in respect of the said common and waste grounds hereby directed to be divided and allotted, and every or any part thereof.

Provided always, and be it further enacted, That nothing herein contained shall extend to extinguish, lessen, or take away any Easter offerings, mortuaries, or surplice fees, belonging or due or payable to James Shylock, rector of B—, or his successors, but the same shall remain due and payable as if this act had not been passed. Surplice fees, &c. reserved.

And be it further enacted, That after the several allotments hereinbefore directed to be made shall be set out, the said commissioners shall and they are hereby authorized and required in the next place to set out, allot and award unto the lord of the said manor of B—, one twentieth part of the residue of the said common or waste grounds hereby directed to be divided and allotted, as a compensation and satisfaction for his estate and interest in and to the soil of all the said common and waste grounds. Allotments to the lord of the manor.

And be it further enacted, That the said commissioners shall and they are hereby required to set out such parts or proportions of the residue of the common and waste grounds intended to be hereby divided and allotted, as in the judgment of the said commissioners shall be equal in value to one half of such residue, which shall be divided and allotted unto and amongst all the several owners and proprietors of ancient messuages, cottages and frontsteads, having right of common upon the said common or waste grounds, according to the number of their respective messuages, cottages and frontsteads; and the said commissioners shall then set out and allot the remaining one half part of such residue of the said common and waste grounds intended to be divided and inclosed, unto and amongst all and every the owners and proprietors of ancient inclosed lands and grounds within the said township of B—, in due pro- Allotment of residue of the commons.

portions, according to the true annual values of such inclosed lands and grounds, such annual values to be ascertained and settled by the said commissioners by the ways and means aforesaid, or such other ways and means as to them shall appear just and equitable.

Tithe allotments to be fenced by the other proprietors.

And be it further enacted, That the several plots or parcels of land or ground hereinbefore directed to be set out, assigned and allotted to the said James Shylock, as such rector, and his successors respectively, in lieu of tythes, shall be inclosed and fenced round with ditches and quickset hedges, or other proper mounds or fences, with proper posts, rails, and other guard fences to such quickset hedges, in such manner as the said commissioners shall order and appoint, by and at the joint expense of all the other proprietors of lands and grounds which shall be exonerated from tythes by virtue of this act, in such manner and proportion as the said commissioners shall order and appoint; and the said commissioners shall direct and appoint to whom the said fences shall belong, after the same shall be properly made as aforesaid; and the said hedges, ditches or other mounds or fences, when properly made and supported as aforesaid, shall be thereafter for ever kept, maintained, supported and scoured by and at the expense of such person or persons to whom the same shall be allotted or directed to belong as aforesaid.

Fencing the other allotments.

And be it further enacted, That the several other allotments to be made of the lands and grounds by this act directed to be divided and allotted, shall within the space of twelve calendar months next after the execution of the said award, or such other time as the said commissioners shall order and direct, be inclosed, hedged, ditched, and fenced at the proper costs and charges of the respective persons to whom the same shall be respectively allotted, or of such other person or persons interested in the said division, (except the rector in right of his rectory) and in such manner as the said commissioners shall order and direct.

Satisfaction for unequal share of fencing.

Provided always, and be it further enacted, That in case through necessity of situation, or any other circumstances, it shall happen that any one or more of the said proprietors shall not have an equal share of mounds or fences allotted to him her or them, it shall and may be lawful to and for the said commissioners, where they shall judge it necessary and reasonable, to award, order, ascertain and appoint, what sum and sums of money such proprietor or proprietors shall pay and

contribute towards making of the mounds and fences of the allotment or allotments of such other proprietor or proprietors, who shall or may have too great a share or proportion of mounding or fencing allotted to him her or them by virtue of this act; and the sum or sums of money so ordered, directed or appointed to be paid, shall be raised, levied, and recovered in such and the like manner as the costs, charges and expenses of obtaining and executing this act can or may be raised, levied and recovered.

And be it further enacted, That it shall be lawful for any person or persons who shall be entitled to any allotment or allotments by virtue of this act, to give, grant, bargain, sell, mortgage, demise, surrender, limit, convey and assure the same, for all or any part of his her or their estate or interest therein, or right thereto, at any time before the execution of the award of the commissioners; and every such gift, grant, bargain, sale, mortgage, demise, surrender, limitation, conveyance and assurance, shall be of the same force and validity as if made after the execution of the said award; and also, that it shall be lawful, for any of the owners or proprietors of any common right upon the lands and grounds hereby directed to be divided and allotted, to sell and dispose of the same, or of the allotment or allotments to be made and set out in respect thereof, by virtue of this act, separately and distinctly from the estate in right of which they he or she are or is entitled to the same, in such manner as they he or she might have done at any time after the execution of the said award.

Proprietors may sell allotments before award, and common rights separately.

And be it further enacted, That it shall and may be lawful to and for the said commissioners to set out, allot and award, any lands, tenements, or hereditaments, in the township of B— aforesaid, in lieu of and in exchange for any other lands, tenements or hereditaments in the same township, or in any adjoining parish, township or place, so that every such exchange be set forth and ascertained by the award of the said commissioners, and also be made with the consent of the respective proprietors of the hereditaments which shall be so exchanged, whether such proprietors shall be bodies politic, corporate or collegiate, corporations aggregate or sole, or tenants in fee simple, fee tail, general or special, or for any life or lives, or for years determinable on any life or lives, and also as to any leasehold hereditaments, by and with the consent of the lessor or lessors of such leasehold hereditaments, and not otherwise;

Exchanges.

or with the consent of the husbands, guardians, trustees, committees, or attornies of or for any such proprietors as aforesaid, who at the time of making such exchanges shall be femmes-coverts, infants, lunatics, persons beyond the seas, or under any other legal disability or incapacity of acting for themselves; such consents to be respectively signified by writing under the hands of the consenting parties, or under the seals of any of them, being corporations aggregate; and every exchange so made and declared shall be for ever good, valid and effectual in law, to all intents and purposes whatsoever; provided, that no such exchange shall be made of any lands, tenements, or hereditaments held in right of any church, chapel, or other ecclesiastical benefice, without consent, testified as aforesaid, of the patron thereof, and of the lord bishop of the diocese in which such lands, tenements or hereditaments, so to be exchanged, shall lie or be situate.

Expences of partitions and exchanges shall be borne by the persons interested.

Provided always, and be it further enacted, That the costs, charges and expenses attending any exchanges or partitions to be made by virtue of this act, or the said recited act of the forty-first year of his present Majesty, shall be borne, paid and defrayed by the several persons interested therein, in such manner and proportions as the said commissioners shall, by writing under their hands, order and direct.

Allotments to be subject to the same settlements as the former estates.

And be it further enacted and declared, That the several lands and tenements which shall be respectively allotted and exchanged by virtue of this act, shall go and remain to the same persons, for the same estates and interests, and to the same uses, upon the same trusts, and to and for the same intents and purposes, and be under and subject to the same powers, provisoes, limitations, conditions, covenants, debts, charges, incumbrances and provisions of every kind, and in the same manner, as the hereditaments in respect whereof the said lands and tenements shall be respectively allotted and exchanged would have belonged, or stood, or been limited to or upon, or been subject unto, if this act had not been passed, (except as to such leases and agreements as shall be determined under this act, or where any other provisions of this or the said recited act shall be to the contrary;) but such allotted and exchanged hereditaments shall nevertheless be subject to such charges and incumbrances thereon as shall be made in pursuance of the said recited act or of this act.

No sheep to

And be it further enacted, That no sheep or lambs shall be

depastured or kept in any of the said intended new inclosures during the space of seven years from the execution of the said award, unless the persons respectively depasturing or keeping sheep or lambs do at their own expense effectually guard and fence the young quickset hedges, or any other person's allotment adjoining to such inclosure respectively, in which sheep or lambs shall be depastured and kept as aforesaid, so as to prevent any damage or injury being done to such fences or quicksets by any such sheep or lambs. be depastured for seven years.

And be it further enacted, That if any of the proprietors or persons interested in the common or waste grounds hereby inclosed, or directed to be divided and allotted, or any other person or persons, shall pay or advance any sum of money in discharge of the fees or other expenses of obtaining or executing this act, the money so paid and advanced shall be repaid and satisfied by the direction of the said commissioners, together with lawful interest for the same. Money advanced for the purposes of this act, to be repaid.

And be it further enacted, That the commissioners shall at all their meetings for putting this act into execution, bear and pay their own expenses; and that when and as soon as the said intended division, and other purposes of this act, shall be finished and completed, and the said commissioners shall have executed their said award, each of the said commissioners shall be respectively paid and allowed after the rate of three pounds and three shillings per day, and no more, for every day upon which he shall have attended a meeting, or transacted business in and for putting this act in execution; and that the surveyor to be appointed by the said commissioners shall be paid such sum or sums of money as the said commissioners shall think proper, for his services and trouble in and about the execution of this act. Allowance to the commissioners.

And be it further enacted, That the costs, charges and expenses preparatory to and of obtaining this act, and of inclosing the allotments which shall by virtue of this act be made to the said James Shylock, in right of his said rectory, and of inclosing the allotments to be made for stone, marle and other public purposes as aforesaid, and all the costs, charges and expenses of surveying, admeasuring, planning, valuing, dividing, and allotting the common and waste grounds to be divided and allotted by virtue of this act, and of preparing the award of the said commissioners, and of the said copies, or duplicates thereof, and all the costs, charges and expenses of the said commis- Expenses of this act, how to be defrayed.

sioners, their assistants and servants, and all other necessary expenses of the several persons to be employed by the said commissioners in and about the premises, either before or after the execution of the said award, and all expenses of forming, completing and repairing the public carriage roads and highways to be set out and appointed by the said commissioners, and all other the expenses of carrying this act into execution, shall be borne and defrayed by the several owners and proprietors of and persons interested in the lands and grounds hereby directed to be divided and allotted, and to whom any allotment shall be made by virtue of this act (save and except the said rector, for or in respect of any allotments made to him as rector, respectively,) which said costs, charges and expenses, together with the proportion thereof to be paid by the several persons hereby made liable to the payment thereof, shall be adjusted by the said commissioners, and shall be paid at such time and place, and to such person or persons, as the said commissioners shall appoint.

Expenses may be raised by sale.

And be it further enacted, That it shall be lawful for the said commissioners and they are hereby required, upon the request of any of the proprietors of the lands hereby directed to be divided and allotted, being tenants in tail, or for life or lives, or for any number of years determinable on a life or lives, or for any other determinable estate or interest, or at the request of the respective husbands, guardians, trustees, committees, agents, or attornies of or for any such proprietors, being under coverture, minors, lunatics, or beyond the seas, or under any other disability or incapacity, to raise the share and proportion, or respective shares and proportions of the person or persons by or on whose behalf any such request shall be made, of the costs, charges and expenses of passing and executing this act, and of ditching, draining, fencing and subdividing their respective allotments, not exceeding the rate of five pounds for every acre of the land allotted to such person or persons by virtue of this act, by the sale of any of the lands or hereditaments to be allotted to such person or persons by virtue of this act, in the manner and subject to the directions prescribed by the said act of the forty-first year of the reign of His present Majesty in that behalf.

Not more than £5. an acre raised by

Provided always, and be it further enacted, That it shall not be lawful for the person or persons from whose allotment land shall be sold as aforesaid, to charge his her or their allotment

or allotments, by virtue of the said recited act or this act, with any money towards the payment of such expenses, unless the money to arise by such sale shall not amount to the sum of five pounds per acre of such allotment or allotments, and then and in such case it shall be lawful for such person or persons to charge his her or their allotment or allotments, or to raise by mortgage thereof, or by such other ways and means as are mentioned in the said recited act or this act, the remainder of such money, for the payment of the expenses of obtaining and executing this act, so as that the whole sum to be raised by virtue of this act shall not exceed the sum of five pounds per acre, according to the number of acres contained in such allotment or allotments. mortgage or sale.

And be it further enacted, That once at least in every year during the execution of this act, (such year to be computed from the passing thereof,) the said commissioners shall and they are hereby required to make a true and just statement or account of all sums of money by them received and expended, or due to them for their own trouble or expenses in the execution of this act; and such statement or account when so made, together with the vouchers relating thereto, shall be by them laid before any one or more of His Majesty's justices of the peace for the —— Riding of the county of York, to be by him or them examined and balanced, and such balance shall be by such justice or justices stated in the books of accounts to be kept by the clerk to the said commissioners, and no charge or item in such accounts shall be binding on the parties concerned, or valid in law, unless the same shall have been duly allowed by such justice or justices. Commissioners to lay accounts before one justice.

And be it further enacted, That the said commissioners shall and they are hereby required to make their award within three years after passing of this act; and in case they shall not make their award within that time, each of the commissioners shall forfeit and lose, out of the allowance hereinbefore directed to be paid to them, the sum of fifty pounds; but, nevertheless, it shall be competent for the said commissioners to make and execute their award at any time after the expiration of the said three years. Award.

And be it further enacted, That within the space of twelve calendar months after the signing and sealing of the award to be made by the said commissioners, a true copy, or duplicate of the same award, written on parchment, and signed and attested by Award to be deposited at the register office.

the said commissioners to be a true copy or duplicate thereof, together with a proper map or plan thereto annexed, shall be delivered to the registrar for the —— Riding of the said county of York, or his deputy, who is hereby required to deposit and keep the same in the register office of the said —— Riding, so that recourse may be had by any person or persons interested in the said premises; for the reception whereof the said registrar, or his deputy, shall be paid the sum of two pounds and two shillings, and no more; and for the inspection or perusal thereof the sum of one shilling, and no more; and the said award shall from and after the delivery of such copy, or duplicate thereof so signed and attested as aforesaid, to the said registrar or his deputy, be deemed and taken to be enrolled according to the direction and within the meaning of the said recited act; and the said award, with a proper map or plan thereto annexed, shall afterwards be deposited in the parish church of B— aforesaid; and the said award, and the said copy, or duplicate thereof, or any other copy thereof, or of any part thereof, attested by the said commissioners, or by the said registrar or his deputy, for which copy no more shall be given than two-pence per sheet, (each sheet containing seventy-two words,) shall be admitted and allowed as legal evidence of the matters and things therein contained, in all courts whatsoever.

Persons aggrieved may appeal to the quarter sessions.

And be it further enacted, That if any person or persons shall think him her or themselves aggrieved by any thing done in pursuance of this or the said recited act, then and in every such case, (except in such cases where the orders and determinations of the said commissioners are by this or the said recited act directed to be final, binding and conclusive, and except in such cases as are hereinbefore directed or authorized to be tried, settled or determined by an action or actions at law,) he she or they may appeal to the general quarter sessions of the peace which may be held in and for the —— Riding of the said county of York, within four calendar months next after the cause of complaint shall have arisen, on giving to the said commissioners, and to the party or parties concerned, ten days notice in writing of such appeal, and the matter thereof, and the justices in their said general quarter sessions are hereby required to hear and determine the matter of every such appeal, and to make such order therein, and to award such costs, as to them in their discretion shall seem reasonable, and by their order and warrant to levy the costs, which shall be awarded by

distress and sale of the goods and chattels of the party or parties liable to pay the same, rendering the overplus, (if any) on demand to the owner or owners of such goods and chattels, after deducting the reasonable charges of such distress and sale, which determination of the said justices shall be final and conclusive to all parties concerned, and shall not be removed or removable by *Certiorari* or any writ or process whatsoever, into any of His Majesty's courts of record at *Westminster*, or elsewhere.

Saving always to the King's Most Excellent Majesty, his heirs and successors, and to all and every person or persons, bodies politic and corporate, his her or their heirs, successors, executors and administrators, all such estate, right, title and interest (other than and except such as is and are hereby meant and intended to be barred, destroyed and extinguished) as they, every or any of them had or enjoyed of, in, to, or out of the said common and waste grounds, so directed to be divided and allotted as aforesaid, before the passing of this act, or could or might have held and enjoyed in case the same had not been made. General saving.

And be it further enacted, That this act shall be printed by the several printers to the King's Most Excellent Majesty, duly authorized to print the statutes of the United Kingdom; and a copy thereof, so printed by any of them, shall be admitted as evidence thereof, by all judges, justices and others. Evidence clause.

Note.—The several clauses contained in the above act, being selected from two acts passed in the year 1817, viz. one for inclosing and exonerating from tithes, a common in the township of Beadlam, in the North Riding of the County of York; and the other for inclosing certain commons and waste grounds in the township of Emley, in the West Riding of the said County, which two acts were drawn up in a very plain and intelligible manner; consequently, I should hope the above will be easily comprehended.—But in order to make every thing as plain and easy to understand as possible, I shall propose the same division to be likewise done by way of a Problem.

PROBLEM.

Let it be required to survey, and afterwards divide the common (See Plate 5,) belonging the township of B, in the county of York, in the following manner, *viz.*— After sufficient highways, gravel pits, &c. are staked out by the commissioners, first allot to the Rev. James Shylock, rector of B——, one sixth part of the value of the common, for his right of tithes thereon; next (after the value of the tithes are deducted) allot to John Blakelock, Esq. lord of the manor, one twentieth part of the remainder for manorial rights; let one half the remainder (after manorial right is deducted,) be equally divided among the frontsteads or common-right houses, having right thereon; and the other half among the proprietors of land in the said township, in proportion to the value of each proprietor's property.

SOLUTION.

Since the method of surveying and planning this, or any other common, is similar to that given in Problem 9, Section 4, Part 2, I shall only give the plan of it; for, by comparing the plan allotted upon, with the above directions, every method used in the division will appear evident.

A LIST OF CLAIMS ALLOWED.

Names of Proprietors.	No. of Fronts Freehold.	No. of Fronts Copyhold.	Quantity Freehold.			Quantity Copyhold.		
			A.	R.	P.	A.	R.	P.
Blakelock, John, Esq.	1		42	3	11			
Day, John	1		0	2	24			
Field, John................	1		0	3	2			
Garth, Thomas............		1				0	1	28
Harrison, John............	1		18	0	29			
Horn, Thomas............	1		23	0	14			
Horswill, Henry..........	1		0	0	31			
Howard, William..........	1		0	0	37			
Jackson, John	1		0	3	25			
Jones, Thomas............	1		1	1	20			
Rose, Peter................	1		1	0	2			
Shylock, Rev. James........	2		27	3	29			
Tasker, Francis............		1				21	1	0

THE

FLAT BOOK.

No. Flat.		A.	Dec.	Value per Acre.	Sh.	Dec.
	James Shylock..	5	356		37	492
1.	Balanced	5	356	*a*	37	492
	John Blakelock..	3	050		21	350
2. North of Road.	Balanced	3	050	*a*	21	350
	Thomas Jones....	2	250		27	000
	James Shylock...	8	511		102	132
3.	Balanced	10	761	*w c*	129	132

No. Flat.	A.	Dec.	Value per Acre.	Sh.	Dec.
Henry Horsewill...	0	022			264
William Howard...	2	424		29	084
John Blakelock.....	9	250		111	004
4. North of Road. Balanced	11	696	*w c*	140	352

Peter Rose	2	331		23	310
Thomas Jones.....	3	105		31	050
James Shylock....	8	342		83	420
5. Balanced	13	778	*w e*	137	780

James Shylock.....	10	754		107	540
6. Balanced	10	754	*w e*	107	540

No. Flat.	A.	Dec.	Value per Acre.	Sh.	Dec.
Peter Rose........	1	721		13	768
James Shylock.....	8	462		67	696
7. Balanced	10	183	*b*	81	464
Francis Tasker.....	9	331		139	965
John Blakelock....	4	617		69	255
James Shylock.....	6	753		101	295
8. Balanced	20	701	*w s*	310	515
Peter Rose........	1	621		24	315
John Jackson......	2	197		32	955
Thomas Jones......		346		5	190
9. Balanced	4	164	*w s*	62	460

No. Flat.	A.	Dec.	Value per Acre.	Sh.	Dec.
Thomas Horn......	1	572		23	580
John Harrison	3	901		58	515
10. Balanced	5	473	*w s*	82	095
Francis Tasker.....	1	806		21	672
John Blakelock....	2	575		30	900
James Shylock.....	3	108		37	296
11. Balanced	7	489	*w c*	89	868
John Blakelock...	6	783		74	613
12. Balanced	6	783	*w w*	74	613

No. Flat.	A.	Dec.	Value per Acre.	Sh.	Dec.
John Harrison......	1	190		13	090
John Blakelock....	2	765		30	415
13. Balanced	3	955	*w w*	43	505
Thomas Garth.....		071		1	207
John Field........		088		1	496
Thomas Horn......	9	498		161	466
John Harrison......	4	186		71	162
14. Balanced	13	843	*w a*	235	331
Thomas Garth.....	3	006		54	108
John Field........	3	132		56	376
John Day..........	3	116		56	088
Thomas Horn......		316		5	688
15. Balanced	9	570	*w b*	172	260

No. Flat.	A.	Dec.	Value per Acre.	Sh.	Dec.
Henry Horswill....	2	958		53	244
John Blakelock....		212		3	816
William Howard . .	1	373		24	714
16. Balanced	4	543	*w b*	81	774
Francis Tasker.....		235		4	230
17. Balanced	0	235	*w b*	4	230
John Jackson......	1	523		27	414
Thomas Jones......		080		1	440
18. Balanced	1	603	*w b*	28	854

FLATS COLLECTED.

No. Ft.	Quantity.		Value per Acre.	Total Value.	
	A.	DEC.		A.	DEC.
1	5	356	*a*	37	492
2	3	050	*a*	21	350
3	10	761	*w c*	129	132
4	11	696	*w c*	140	352
5	13	778	*w e*	137	780
6	10	754	*w e*	107	540
7	10	183	*b*	81	464
8	20	701	*w s*	310	515
9	4	164	*w s*	62	460
10	5	473	*w s*	82	095
11	7	489	*w c*	89	868
12	6	783	*w w*	74	613
13	3	955	*w w*	43	505
14	13	843	*w a*	235	331
15	9	570	*w b*	172	260
16	4	543	*w b*	81	774
17	0	235	*w b*	4	230
18	1	603	*w b*	28	854
	143	937		1840	615

EXPLANATION OF THE FLAT-BOOK.

After the book is divided and ruled in columns, each flat is then entered, with its quantity, value per acre, and total value, in one line, as above, with a double line ruled over the quantity, &c. The blank parts are filled up, as the surveyor allots, by entering into them the person's name, with the quantity and value allotted to him from that particular flat; so that the book shews at once, how and in what manner each flat is disposed of. Due care must be taken, when the whole of each flat is allotted, to make a remark that the flat is truly balanced.

Note.—It is to be observed, that the remainder, or quantity and value of the last piece allotted, in every flat, found in the above book, by taking the difference between the quantity and value allotted before, and the quantity and value of the whole flat, which saves the trouble of keeping a profit and loss account, as I shall shew hereafter.

THE

DEDUCTION SHEET.

Proprietors' Names.	(1.) Value of Ancient Inclosure.		(2.) No. of common-right Houses.	(3.) Common for Land.		(4.) Common for Houses.		(5.) 3rd and 4th col. added.		(6.) $\frac{1}{6}$ of last col. for Tythes.		(7.) 5th and 6th col. substracted.		(8.) $\frac{1}{20}$ of last col. for Manorial Right.		(9.) 6th and 8th col. added.		(10.) 9th and 5th substracted, gives the neat property to be allotted.	
	sh.	dec. (.4840758)		sh.	dec.	sh.	dec.	sh.	dec.	sh.	dec.	sh.	dec.	sh.	dec.	sh.	dec.	sh.	dec.
Blakelock, John .	554	750	1	268	541	65	736	334	277	55	713	278	564	13	9282	69	641	264	636
Day, John.......	10	583	1	5	123	65	736	70	859	11	809	59	050	2	9525	14	761	56	098
Field, John......	15	250	1	7	382	65	736	73	118	12	186	60	932	3	0466	15	232	57	886
Garth, Thomas ...	8	583	1	4	154	65	736	69	890	11	648	58	242	2	9121	14	560	55	330
Harrison, John ..	236	833	1	114	645	65	736	180	381	30	063	150	318	7	5159	37	578	142	803
Horn, Thomas ...	361	916	1	175	194	65	736	240	930	40	155	200	775	10	0387	50	193	190	737
Horswill, Henry ..	3	833	1	1	855	65	736	67	591	11	265	56	226	2	8163	14	081	53	510
Howard, William .	4	583	1	2	218	65	736	67	954	11	325	56	529	2	8314	14	156	53	798
Jackson, John . ..	21	750	1	10	528	65	736	76	264	12	710	63	554	3	1777	15	887	60	377
Jones, Thomas ...	33	000	1	15	974	65	736	81	710	13	618	68	092	3	4046	17	022	64	688
Rose, Peter......	24	333	1	11	779	65	736	77	515	12	919	64	596	3	2298	16	148	61	367
Shylock, James	328	833	2	159	180	131	472	290	652	48	442	242	210	12	1105	60	552	230	100
Tasker, Francis ..	296	916	1	143	729	65	736	209	465	34	911	174	554	8	7277	43	638	165	827
	1901	163	14	920	302	920	304	1840	606	306	764	1533	842	76	695	383	449	1457	157

Column eighth, Manorial Right 76 695
Column sixth, Tythes 306 764

Total Value of the Flats or Common to be allotted........1840 616

EXPLANATION OF THE DEDUCTION SHEET.

In the first column, opposite each proprietor's name, is put down the value of his ancient inclosed ground, taken from the Survey-book, Section 6. In the second column, is put down his number of fronts or common-right houses, taken from the list of claims* allowed by the commissioners. In the third, is put the value of the common, due to him for land, which is found by multiplying his property by .4840758, which multiplier is found by the rule of Fellowship, according to the above directions, and is equal to half the value of the common, divided by the value of the ancient inclosed land. In the fourth, is put the value of the common due to him for his fronts. For an explanation of the other columns, see their respective heads.

By what I have said respecting the four first valuable columns, and how I have found that the 6th, 8th, and 10th columns, or the tythes, manorial right, and value to be allotted for houses and land, when added together, are equal to the value of all the flats (or common) to be divided, the truth of the deductions is manifest. A plan of the new roads is made as soon as the commissioners think convenient, and is proceeded with in the same manner as the claims. Those who wish to see more on this head, I refer to the

* As soon as the commissioners have taken the oath to capacitate them to act, each proprietor gives to them an account in writing, (signed by his or her name) containing the quantity of old inclosure, and the number of common-right houses, with their respective tenure, for which he or she claims to be intituled to a right of common; which account is called his or her claim. A list is then made out, containing the whole of them, and put upon the door of the parish church the township lays in; a meeting is likewise advertised to hear objections to them, if any; the objected claims are again put upon the church door, and another meeting advertised to hear evidence, and to allow or disallow the said objected claims, according to the evidence there given.

general inclosure act, where every proceeding is fully explained.

The deduction sheet being made out, we next, according to the directions, proceed to allot to each proprietor his respective share of the common, as in the following

EXAMPLE.

Required the quantity and value in each flat, when the sum of the said values is equal to Thomas Garth's claim in the deduction sheet, or 55.330 shillings.

First, after determining the line A B, (see Fig. Plate 5) so that the value contained in the part A B F G is equal the value of Thomas Garth's, John Field's, and John Day's claims, when added together; draw the guess line $e f g$, and find the separate areas of $e f h G$, and $f g A h$, by the directions laid down in Section 2, Part 2, viz. $e f h G$, part of flat 15, = 2.836; and $f g A h$, part of flat 14, = .065; and as the value per acre of flat 15 and 14 is eighteen and seventeen shillings, the operation will stand as follows :—

Flat.	Quantity.	s.	Value.
15 =	2.836 ×	18 =	51.048
14 =	.065 ×	17 =	1.105

Value 52.153, laid off by guess.

Thomas Garth's claim, by the deduction sheet, is 55.330
Value laid off by guess . . 52.153

Value to be added 3.177

Now, by the scale planned from, we have $e f = 850$, and $f g = 30$ links; hence,

$$\text{by Prob. 2, Sec. 7,}\left\{\begin{array}{r} 850 \times 18s. = .15300 \\ 30 \times 17s. = .00510 \end{array}\right\}\begin{array}{l}\text{The length mul-} \\ \text{tiplied by the} \\ \text{value;}\end{array}$$

$$.15810$$

and $\frac{3.17700}{.15810} = 20$ links $= c\ e = g\ D$, the breadth to be

added; consequently, $850 \times 20 = .170$, the area of $Cefn$;
and $30 \times 20 = .006$, the area of $fnDg$;
which two areas, added to those laid off by guess, will be the whole area of his allotment. Therefore,

$.065 + .006 = .071$ at 17*s.* $= 1.207$, the quantity and value in flat 14;
and $.170 + 2.836 = 3.006$ at 18*s.* $= 54.108$, the quantity and value in flat 15:

Hence, 3.077 and 55.315 is the whole quantity and value laid off,—which value is nearly equal the value to be allotted him in the deduction sheet; consequently, the two separate quantities and values of each flat are next entered into the flat-book, in their respective flats, as well as into the allotment-book, opposite his respective claim; the allotment marked off, with ink, upon the plan; his name and quantity inserted, and it is done—excepting the fences, and subdividing or allotting the different tenures, which see below. And thus we proceed with each proprietor's allotment, until the whole is completed.

Note 1.—The value of the part A B F G, is allotted exactly in the above manner, before we proceed to allot to T. Garth.

Note. 2.—To make the allotment as commodious as we can, it is the practice to allot two, three, or more small allotments together; after that, to subdivide them as above.

Note 3.—It is customary, in most inclosures of a common, to allot to all the small proprietors, in as convenient places as can be found, their respective shares of the common, in the first place; after that, to collect the remaining flats, and pieces of flats, in different parts, and compare their value with the value to be allotted to the great proprietors, before beginning to allot them their respective shares, so as to have a check upon the proceedings, as well as to indulge each proprietor, by laying out the whole or part of his allotments in such place or places as will be most convenient and eligible;—which *method* I shall apply to the division now treating of, as follows:—After allotting to Thomas Garth, John Field, &c. &c. their respective allotments, collect the several flats, and parts, or remainders, un-

allotted, and compare them with the sum of John Blakelock's and James Shylock's claims, in the deduction sheet; if they agree, then look for their conveniences, where you will readily find, that flat 2nd, and part of flat 4th, adjoin on John Blakelock's ancient inclosure,—therefore allot him the said flat and parts; also allot him part of flat 13th. After that, sum up the value allotted to him, and subtract it from the value of his claim, due to him at first, and it will give you the value he still wants; which value you must allot him elsewhere, as from flats 8th, 11th, and 12th; and insert their respective parts in the flat and allotment books; so that, in the three separate allotments, he will get the whole value due to him for houses, lands, and manorial right: the residue of the common unallotted, then falls to the Rev. James Shylock, to whom you must allot it, after being duly examined, and enter each flat, or part of flat, into the flat and allotment book accordingly. In the next place, proceed to subdivide the allotments set out and allotted to the said John Blakelock and James Shylock, according to their respective tenures; that is, to John Blakelock, Esq. the value of his manorial right, from and of such part of his said allotments as you think will suit best, and insert his name, tenure, and quantity in the allotment, upon the plan. After that, allot to the Rev. James Shylock, from and off the allotments set out for him, the full value to be allotted for his freehold land and houses, according to the deduction sheet, and insert his name, tenure, &c. in the allotment likewise. Lastly, according to the best of your skill, mark upon the plan, what fences the owners and occupiers of each allotment are to make, and afterwards keep in repair; and so you finish the whole allotting upon the plan.

Note 4.—It will frequently happen in inclosures, that a proprietor is possessed of houses, lands, &c. of two or three tenures, as freehold, copyhold, and leasehold; now, if his whole claim be laid out or allotted in one allotment, the said allotment must be subdivided as above.

Note 5.—In subdividing, no regard is paid to the flat book, *i. e.* no flats or parts of flats are ever entered therein; being chiefly done upon loose sheets of paper.

Note 6.—After finishing allotting upon the plan, &c. next proceed to transfer the allotments to the ground to be divided, with as much care as you possibly can, in the following manner:—First, take a rough sketch of all the allotments, lanes, ⊙s, and fixed marks, from the plan allotted upon; then, with the scale plotted from, measure in a direct line from some fixed ⊙ or point on the plan, to some other fixed ⊙ or point—as from G, the north extremity of the east fence, bounding the Near New-Lane, to F., the south extremity thereof—and note down upon the sketch, when proportioned as below, the distance from the ⊙ or point measured from, to the crossing or abutting of the first allotment fence,—the distance between the first and

second fence,—and the distance between the second and third (or the point F.) Next, draw a line upon the plan, from the point F to some other fixed point, at or near the gate at the east end of the A and P road; produce the fence of each allotment, until it meet the said line; then measure the respective distances between the point of intersection, proportion them, and put them down upon your sketch as before.—Again, to determine the other ends of the fences belonging to each respective allotment, draw the line between ⊙ 20 and ⊙ 24, measure the respective distances on each side of the fence crossed, which distances proportion, and note down upon your sketch likewise; after that, from ⊙ 21, or some fixed place near it, draw a line to ⊙ 26; likewise measure, proportion, and note down upon your sketch, the respective distances, in the same manner. Lastly, from the point of intersection, near the point A, on the line A B, measure, proportion, and put upon your sketch, the breadth of each allotment abutting thereon; then have you got the dimensions for staking out all the allotments bounded by the Near New Road, on the west, and the A and P road on the south; and in the very same manner you must proceed, from place to place upon the plan allotted upon, until you have got proper dimensions for staking out the whole of the allotments the common is to be divided into. Next, take your chain and measure the said distances upon the ground, (beginning and proceeding line after line, as you did with the plan,) and put a stake into the ground at the extremity or end of every fence that crosses or abuts upon the line, measuring, as at the south-west corners of Thomas Garth's and John Field's allotments: but after you have put down a stake in the corner of the last mentioned allotments, (although the point F is fixed,) measure from thence to F, and see if the distance measured upon the ground agrees with that you have upon your sketch; if it does, the extremities of the fences abutting upon that line are truly determined; if not, you must then, by the Rule of Proportion, say, as the sum of the distances taken from the plan is to the sum of the distances measured upon the ground, so is each distance taken from the plan to its true distance to be laid off upon the ground: next, remove the stakes into their true places; after that, proceed line after line, in the same manner, until you have determined the position of all the fences to be staked out upon the common; observing always, when the extremities of one or more lines are determined, to stake them out as you proceed in the business. The method of staking out the lines is very simple; for by fixing a pole at one extremity, you have nothing more to do than stand at the other, and direct the person who carries the stakes, to put one into the ground in a line with you and the pole, at the distance of 20 or 30 yards from you, another at the same distance from the last put down, &c. to the end of the line; observing also to have three or four stakes between you and the person who carries them; that the person who drives them into the ground keep them always perpendicular to the horizon; and that double stakes be put down at the extremity of each fence or line staked.

Note 7.—It is very well known, that in taking the length of a line upon the plan, with a scale of three or four chains to an inch, no person can judge of its length to a link; consequently, in taking a number of breadths or distances, between one division fence and another, the error will multiply, and the sum of the breadths so taken will not agree with the whole line measured on the plan. Now, as the true length of the whole line can be ascertained upon the plan, as near as any small part of it, the necessity of proportioning them before they are laid down upon the sketch, is evident. The necessity of proportioning them in the field is likewise obvious; for it is very rare that a line laying between two field points, determined when surveying, will measure the same length upon the plan, when allotting or staking, as it will upon the ground, by reason of the contracting and expanding of the paper.

THE

ALLOTMENT BOOK.

	Value to be Allotted.	
	s.	dec.
JOHN DAY.		
For House and Land.	56	098
JOHN FIELD.		
For House and Land.	57	886

No. Ft.	Acres.	Dec.	Value per Acre.	Total Value.	
				s.	dec.
15	3	116	*w b*	56	088
14		088	*w a*	1	496
15	3	132	*w b*	56	376
	3	220		57	872

	Value to be Allotted.	
	s.	dec.
THOMAS GARTH.		
For House and Land......	55	330
THOMAS HORN.		
For House and Land......	190	737

No. Ft.	Acres.	Dec.	Value per Acre.	Total Value.	
				s.	dec.
14		071	*w a*	1	207
15	3	006	*w b*	54	108
	3	077		55	315
10	1	572	*w s*	23	580
14	9	498	*w a*	161	466
15		316	*w b*	5	688
	11	386		190	734

	Value to be Allotted.	
	s.	dec.
JOHN HARRISON.		
For House and Land	142	803
HENRY HORSWILL.		
For House and Land	53	510

No. Ft.	Acres.	Dec.	Value per Acre.	Total Value. s.	Total Value. dec.
10	3	901	*w s*	58	515
13	1	190	*w w*	13	090
14	4	186	*w a*	71	162
	9	277		142	767
4	0	022	*w c*		264
16	2	958	*w b*	53	244
	2	980		53	508

	Value to be Allotted.	
	s.	dec.
WILLIAM HOWARD.		
For House and Land......	53	798
PETER ROSE.		
For House and Land......	61	367

No. Ft.	Acres.	Dec.	Value per Acre.	Total Value.	
				s.	dec.
4	2	424	*w c*	29	088
16	1	373	*w b*	24	714
	3	797		53	802
5	2	331	*w e*	23	310
7	1	721	*b*	13	768
9	1	621	*w s*	24	315
	5	673		61	393

	Value to be Allotted.	
	s.	dec.
JOHN JACKSON.		
For House and Land......	60	377
THOMAS JONES.		
For House and Land......	64	688

No. Ft.	Acres.	Dec.	Value per Acre.	Total Value.	
				s.	dec.
9	2	197	*w s*	32	955
18	1	523	*w b*	27	414
	3	720		60	369

No. Ft.	Acres.	Dec.	Value per Acre.	s.	dec.
3	2	250	*w c*	27	000
5	3	105	*w e*	31	050
9		346	*w s*	5	190
18		080	*w b*	1	440
	5	781		64	680

	Value to be Allotted.	
	s.	dec.
FRANCIS TASKER.		
For House and Land	165	827
JOHN BLAKELOCK, Esq.		
For Manorial Right.	76	695
For House and Land	264	636
	341	331

No. Ft.	Acres.	Dec.	Value per Acre.	Total Value. s.	Total Value. dec.
8	9	331	*w s*	139	965
11	1	806	*w c*	21	672
17		235	*w b*	4	230
	11	372		165	867

2	3	050	*a*	21	350
4	9	250	*w c*	111	000
8	4	617	*w s*	69	255
11	2	575	*w c*	30	900
12	6	783	*w w*	74	613
13	2	765	*w w*	30	415
16		212	*w b*	3	816
	29	252		341	349

	Value to be Allotted. s.	dec.
The Rev. JAMES SHYLOCK, Rector.		
For House and Land	230	100
For Tythes.	306	764
	536	864

GENERAL

	Acres.	Dec.	s.	Dec.
The original quantity and value of the whole Common, after the Roads and Gravel Pits were set out, was	143	937	1840	615
Roads	9	950		
Quarries of Gravel Pits	1			
	154	887		

No. Flat.	Acres.	Dec.	Value per Acre.	s.	Dec.
1	5	356	*a*	37	492
3	8	511	*w c*	102	132
5	8	342	*w e*	83	420
6	10	754	*w e*	107	540
7	8	462	*b*	67	696
8	6	753	*w s*	101	295
11	3	108	*w c*	37	296
	51	286		536	871

BALANCE.

THE QUANTITY AND VALUE

Of all the Allotments set out or allotted to the Proprietors, viz.

	Acres.	Dec.	s.	Dec.
John Day	3	116	56	088
John Field	3	220	57	872
Thomas Garth	3	077	55	315
Thomas Horn	11	386	190	734
John Harrison	9	277	142	767
Henry Horswill	2	980	53	508
William Howard	3	797	53	802
Peter Rose	5	673	61	393
John Jackson	3	720	60	369
Thomas Jones	5	781	64	680
Francis Tasker	11	372	165	867
John Blakelock	29	252	341	349
James Shylock	51	286	536	871
	143	937	1840	615
Roads	9	950		
Quarries	1			
	154	887		

EXPLANATION OF THE ALLOTMENT-BOOK.

This book is the same as a ledger in Book-Keeping, being divided into columns, and composed of Debtor and Creditor accounts, kept with each proprietor, as you see above; and is opened along with the flat-book, when you begin to allot; making yourself to each proprietor Debtor, for his several claims in land, houses, &c., as well as Creditor for the several flats, and pieces of flats, that have been allotted to him, and entered in the flat-book; so that when the value of the several flats, or pieces of flats, or both, allotted to each proprietor, becomes equal to his respective claim or claims, and the whole value of the pieces each flat is divided into becomes equal to its respective whole, in the flat-book, it is evident the common must be divided without error.

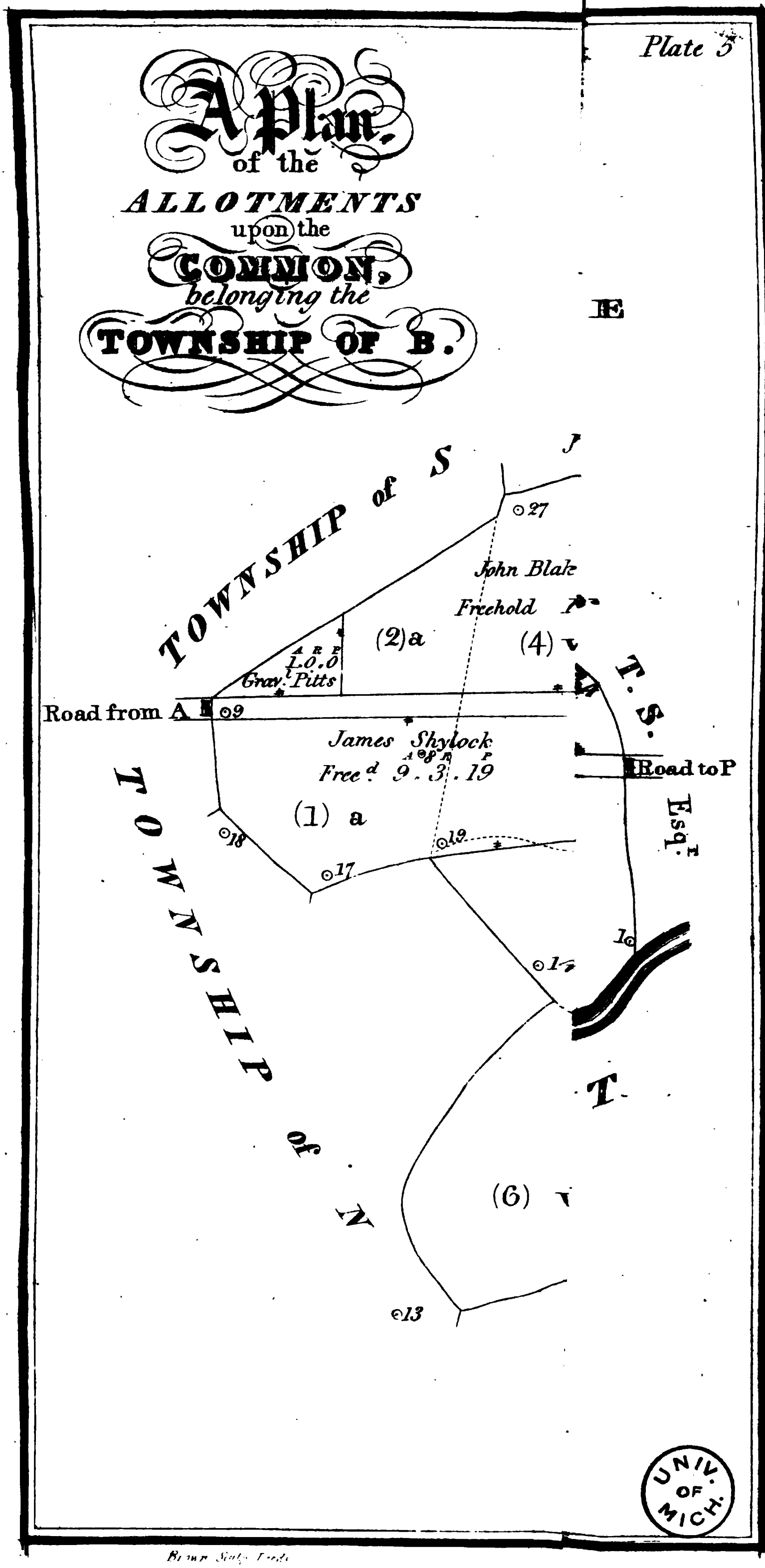
I have now fully explained the books, &c. and also shewn that the general balance, after finishing allotting, agrees to the last place of decimals—(a case which should and will always happen, when the allotment, set out for each proprietor, agrees nearly with his net claim brought out in the deduction sheet)—by reason of the remainders (or last piece of every flat allotted) being taken equal to the difference between the parts allotted before and the whole, as I have done here,—a method which is likewise practised by a great number of surveyors, and will vary very insensibly from the truth, provided the quantity and value of the whole flat, in the first place, be accurately found and laid down, as well as the other quantities and values parted from it afterwards.

But when the inclosure is a large one, in order that the Commissioners may make allowance for openings, &c. the most safe and correct method is to make a reserve of a few shillings-worth of land, before the deduction sheet is made

out, and cast every remainder; so keep a profit and loss account, to shew how the reserve is disposed of, and what is gained or lost in every flat by dividing it; which profit and loss account, with the value of the flats and allotments, will, if the division be right, make a balance similar to the above.

The common being now divided upon the plan, the fences noted, each proprietor's name, with the tenure, and quantity in acres, roods, and perches, put into his allotment, and the allotments transferred from the plan to the ground, by Note 6, Page 240; next, draw up notices, and send one to each proprietor, to let him or her know on what part of the common his or her allotment lays, and what it contains in acres, roods, and perches; also whose allotments or lands it is bounded by on the east, west, north, and south; and likewise what fences he is to make, and afterwards keep in repair. Lastly, instructions must be given to the Solicitor, for drawing up the award, (as will be seen in the following); and to accompany it, the reduced plan, Plate 6, must be copied from the original, and similar in respect to every remark made thereon, excepting the quality lines, number of the flat, value per acre, and stations, which may be left out, being of no more use; then the division is completed.

Plate 5


A Plan,
of the
ALLOTMENTS
upon the
COMMON,
belonging the
TOWNSHIP OF B.
TOWNSHIP of S
TOWNSHIP of N
John Blak
Freehold
(2)a
(4)
A R P
1.0.0
Grav. Pitts
Road from A
9
27
James Shylock
Free.d 9.3.19
(1) a
18
17
19
T. S.
Road to P
Esq.r
T
(6)
13


6
A PLAN,
TO
ACCOMPANY
the
Award.
T O W

AWARD OF THE COMMISSIONERS

IN

THE INCLOSURE OF WASTE LANDS OR COMMON,

Within the Parish of B—, in the County of York.

THE AWARD.

To all to whom these Presents shall come.

T. H. of N—, in the county of York, gent., and R. B. of W—, in the said county of York, gent., send greeting:—

Whereas in and by an act of Parliament, made and passed in the —— year of the reign of his present Majesty King George the Fourth, intituled "An act for inclosing and exonerating from tythes, lands in the parish of B—, in the —— Riding of county of York;" it was amongst other things enacted, that we, T. H. and R. B. should be, and we were thereby appointed commissioners for dividing, allotting, and inclosing the said lands, and for carrying the said act into execution.

And whereas we the said T, H. and R. B. did take and subscribe the oath required by the act of the 41st of George III. intituled, "An act for consolidating in one act, certain provisions, usually inserted in acts of inclosures, and for facilitating the mode of proving the several facts usually required in the passing of such acts; and that we the said T. H. and R. B. did cause such notice to be given of our first and other meetings, as is by the said acts required; and did nominate and appoint E. C. of D—, in the county of York, gent. to be umpire, touching any thing relating to the said inclosure; and proceeded to divide, set out and allot, the said waste lands or common, in execution of the several powers and authorities, by the said first herein-before-recited act in us reposed.

Now know ye, that we the said T. H. and R. B. having duly weighed and considered all and every the matters and things referred to our determination, judgment and arbitration, do, on the day of the date hereof, by virtue and in exercise of the powers, trusts, and authorities vested in or given to us, in and by the said herein-before-recited acts of Parliament, and of all

other powers and authorities whatsoever, to us in any wise belonging or appertaining, make, publish, and declare this our award, order, and determination, touching and concerning the premises in manner and form following,—that is to say, we declare that the whole of the waste lands or common by the said recited act, of the —— year of the reign of his present Majesty, directed and intended to be divided and inclosed, contained in statute measure, 154 A. 3 R. 22 P., and that the several allotments into which the said waste lands or common are divided, and which are herein-after by us awarded to the parties interested therein, do respectively contain in statute measure, be the same more or less, the quantity in acres, roods, and perches, herein-after, in this our award, specified.

Stone quarry or gravel pits, 1 A. 0 R. 0 P.

And we do, in pursuance of the special directions of the said first herein recited act, allot, appoint, and award unto the surveyors of the highways for the time being, of the township of B—, for the purpose of getting stone or gravel for repairing the public and private roads within the said township of B—, and also to be used in common by the proprietors of lands and estates in the said township, and by their tenants, for their own necessary uses and purposes, in and about the repairs of estates in the said township, all that piece or parcel of land, as the same is now set out, ditched or bounded, containing 1 A. 0 R. 0 P. lying at the west end of the said common, adjoining on an allotment herein awarded to John Blakelock, Esq. on the east, on the A. and P. road on the south, and on the township of S— on the west and north; and we order and direct that the surveyors of the said highways for the time being, shall make, and for ever maintain a sufficient ditch and fence on the south and east side thereof. We do also, in obedience to the said act, appoint and award, that there shall be one public carriage road and highway, of the width of 66 feet, between and exclusive of the ditches or fences as and where the same is now made and used, called the A. and P. road, beginning at a place called —— Gate, and extending from thence eastward, in the direction of the ancient road, to a place called —— Gate, leading to the township of P.

A. and P. road.

We do also, in obedience to the said act, appoint and award, that there shall be one other public carriage road and highway, of the width of 66 feet, between and exclusive of the ditches or fences, as and where the same is now made and used, called the Far New Road, branching out of the last mentioned road, and

Far New Road.

extending southward to the south-west corner of an allotment herein awarded to Francis Tasker,—from thence in a south-west direction to an ancient lane in the township of H—.

Near New Road.

We do also, in obedience to the said act, appoint and award that there shall be one other public carriage road and highway, of the width of 66 feet, between and exclusive of the ditches or fences, as and where the same is now made and used, called the Near New Road, branching out of the said A. and P. road, and extending northwards to an ancient lane leading to the town of B—. And we order and direct that the three public carriage roads, last herein-before respectively awarded, shall be for ever hereafter maintained and repaired by the occupiers of lands, tenements, and hereditaments in the township of B—.

James Shylock, rector, owner of the tithes for the time being, 31 A. 2 R. 9 P.

We also allot, appoint, and award to the Rev. James Shylock, rector, and owner of the tythes, for the time being, all that piece or parcel of land upon the aforesaid common, containing 31 A. 2 R. 9 P., adjoining on the allotments herein awarded to Thomas Jones, Peter Rose, and Far New Lane on east, on the township of H— on the south, on the township of N—, and on an allotment herein awarded to the said James Shylock on the west, and on the allotments herein awarded to Peter Rose, James Shylock, and the A. and P. road on the north; and we order and direct that the said James Shylock, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain a sufficient ditch and fence on the east and north sides thereof; (excepting so much on the north as adjoins the allotment herein awarded to the said James Shylock as freehold of inheritance) such ditch and fence being first made and formed under the directions of the said first recited act.

John Blakelock, lord of the manor of B—, 6 A. 3 R. 25 P.

We also allot, appoint, and award unto John Blakelock, Esq. as lord of the manor of B—, all that piece or parcel of land in the aforesaid common, containing 6 A. 3 R. 25 P., adjoining on the A. and P. road on the north, on lands belonging T. S., Esq. on the east, on the river T. on the south, and on an allotment herein awarded to the said John Blakelock, Esq. on the west; and we order and direct that the said John Blakelock, Esq. and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain a sufficient ditch and fence on the west and north sides thereof.

Allotment in right of frontsteads, &c., John

We also allot, appoint, and award unto John Blakelock, Esq. all that piece or parcel of land on the said moor, containing 7 A. 0 R. 11 P., adjoining the A. and P.

Blakelock, 7 A. 0 R. 11 P. road on the north, on an allotment herein awarded to James Shylock on the west, on the river T. on the south, and on an alloment herein awarded to the said John Blakelock, as lord of the manor of B— on the west; and we order and direct that the said John Blakelock, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain a sufficient ditch and fence on the west and north sides thereof.

2 A. 3 R. 2 P. We also allot, appoint, and award to the said John Blakelock, all that piece or parcel of land in the moor, containing 2 A. 3 R. 2 P., adjoining on the A. and P. road on the south, on lands belonging T. S. Esq. on the east and north, and on an allotment herein awarded to John Harrison on the west; and we order and direct that the said John Blakelock, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain a sufficient ditch and fence on the west and south sides thereof.

12 A. 2 R. 1 P. We also allot, appoint, and award to the said John Blakelock, all that piece or parcel of land in the said common, containing 12 A. 2 R. 1 P., adjoining on the A. and P. road on the south, on an allotment herein awarded to the surveyors of the highways on the west, on an allotment herein awarded to Henry Horswill and to William Howard on the east, and partly on the township of S—, and partly on old inclosed lands, belonging the said John Blakelock, on the north; and we order and direct that the said John Blakelock, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the south and east sides thereof. All which said several allotments herein awarded to the said John Blakelock, shall be held as freehold of inheritance.

John Day, 3 A. 0 R. 18 P. We also allot, appoint, and award unto John Day, all that piece or parcel of land in the common, containing 3 A. 0 R. 18 P., adjoining on the A. and P. road on the south, on the Near New Road on the west, on an allotment herein awarded to John Field on the north, and on an allotment herein awarded to Thomas Horn on the east; and we order and direct that the said John Day, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain a sufficient ditch and fence on the west and south sides thereof. Which said allotment to the said John Day, shall be held as freehold of inheritance.

John Field, 3 A. 0 R. 35 P. We also allot, appoint, and award to John Field, all that piece or parcel of land in the common, containing 3 A. 0 R. 35 P., adjoining an allotment herein awarded to John Day on the south, on the Near New

Road on the west, on an allotment herein awarded to Thomas Garth on the north, and on an allotment herein awarded to Thomas Horn on the east; and we order and direct that the said John Field, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain a sufficient ditch and fence on the south and west sides thereof. Which said allotment to the said John Field shall be held as freehold of inheritance.

Thomas Garth, 3 A. 0 R. 12 P.

We also allot, appoint, and award to Thos. Garth, all that piece or parcel of land in the common, containing 3A. 0R. 12P., adjoining on an allotment herein awarded to John Field on the south, on the Near New Road on the west, on ancient inclosures, belonging Thomas Horn, on the north, and on an allotment herein awarded to Thomas Horn on the east; and we order and direct that the said Thomas Garth, and the owners and occupiers for the time being, of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the west and south sides thereof; which said allotment, to the said Thomas Garth, shall be held as copyhold of the manor of ———.

John Harrison, 9 A. 1 R. 4 P.

We also allot, appoint, and award to John Harrison, all that piece or parcel of land in the common, containing 9A. 1R. 4P., adjoining partly on an allotment herein awarded to John Blakelock, and partly on lands belonging T. S. Esq., on the east, on the A. and P. road on the south, on an allotment herein awarded to Thomas Horn, on the west, and on ancient inclosed lands, belonging the said John Harrison, on the north; and we order and direct that the said John Harrison, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the south and west sides thereof; which said allotment, to the said John Harrison, shall be held as freehold of inheritance.

Thomas Horn, 11 A. 1 R. 21 P.

We also allot, appoint, and award to Thomas Horn, all that piece or parcel of land in the common, containing 11A. 1R. 21P., adjoining on the A. and P. road on the south, on allotments herein respectively awarded to John Day, John Field, and Thomas Garth, on the west, on ancient inclosed lands belonging the said Thomas Horn, on the north, and on an allotment herein awarded to John Harrison, on the east; and we order and direct, that the said Thomas Horn, and the owners and occupiers for the time being, of this allotment, shall make, and for ever maintain a sufficient ditch and fence on the south and west sides thereof; which said allotment, to the said Thomas Horn, shall be held as

freehold of inheritance. We also allot, appoint, and award to Henry Horswill, all that piece or parcel of land in the common, containing 2A. 3R. 37P., adjoining on the Near New Road on the east, on an allotment herein awarded to William Howard on the south, on an allotment herein awarded to John Blakelock on the west, and on ancient inclosed lands, belonging the said John Blakelock on the north; and we order and direct, that the said Henry Horswill, and the owners and occupiers for the time being, of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the east and south sides thereof; which said allotment to the said Henry Horswill, shall be held as freehold of inheritance. We also allot, appoint, and award to William Howard, all that piece or parcel of land in the common, containing 3A. 3R. 7P., adjoining on the A. and P. road on the south, on the Near New Road on the east, on an allotment herein awarded to Henry Horswill on the north, and on an allotment herein awarded to John Blakelock on the west; and we order and direct, that the said William Howard, and the owners and occupiers for the time being, of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the south and east sides thereof; which said allotment to the said William Howard, shall be held as freehold of inheritance. We also allot, appoint, and award to John Jackson, all that piece or parcel of land in the common, containing 3A. 2R. 35P. adjoining on the A. and P. road on the north, on the Far New Road on the east, on an allotment herein awarded to Peter Rose on the south, and on an allotment herein awarded to Thomas Jones on the west; and we order and direct that the said John Jackson, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the north and east sides thereof; which said allotment to the said John Jackson shall be held as freehold of inheritance.

Henry Horswill, 2A. 3R. 37P.

William Howard, 3A. 3R. 7P.

John Jackson, 3A. 2R. 35P.

We also allot, appoint, and award to Thomas Jones, all that piece or parcel of land, in the common, containing 5A. 3R. 5P. adjoining on the A. and P. road on the north, on an allotment herein awarded to John Jackson on the east, on an allotment herein awarded to Peter Rose on the south, and on an allotment herein awarded to the Rev. James Shylock, rector, (as owner of the tithes) on the west; and we order and direct that the said Thomas Jones, and the owners and occupiers for the time being of this allotment, shall make, and for ever main-

Thomas Jones, 5A. 3R. 5P.

tain a sufficient ditch and fence on the north and east sides thereof; which said allotment to the said Thomas Jones shall be held as freehold of inheritance.

Peter Rose, 5A. 2R. 27P. We also allot, appoint, and award to Peter Rose, all that piece or parcel of land in the common, containing 5A. 2R. 27P. adjoining on the Far New Road on the east, on an allotment herein awarded to James Shylock (as owner of the tithes) on the south and west, and on the allotments respectively awarded to Thomas Jones and John Jackson on the north; and we order and direct that the said Peter Rose, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the east and north sides thereof; which said allotment to the said Peter Rose shall be held as freehold of inheritance.

James Shylock, 9A. 3R. 17P. We also allot, appoint, and award to the Rev. James Shylock, all that piece or parcel of land in the common, containing 9A. 3R. 17P. adjoining on an allotment herein awarded to John Blakelock on the east, on the A. and P. road on the north, on an allotment herein awarded to Francis Tasker on the west, and partly on the township of H——, and partly on the river T. on the south; and we order and direct that the said James Shylock, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the north and west sides thereof.

9A. 3R. 19P. We also allot, appoint, and award unto the said James Shylock, all that piece or parcel of land in the common, containing 9A. 3R. 19P. adjoining on an allotment herein awarded to the said James Shylock (as owner of the tithes) on the east, on the A. and P. road on the north, on the township of N—— on the west, and partly on the said township of N——, and partly on an allotment herein awarded to the said James Shylock (as owner of the tithes) on the south; and we order and direct that the said James Shylock, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain, a sufficient ditch and fence on the north, and as much of the south side thereof as adjoins his said tithe allotment; which two last allotments herein awarded to the said James Shylock shall be held as freehold of inheritance.

Francis Tasker, 11A. 1R. 19P. We also allot, appoint, and award to Francis Tasker, all that piece or parcel of land in the common, containing 11A. 1R. 19P. adjoining on the A. and P. road on the north, on the Far New Road on the west, on the township of H—— on the south, and on an allotment herein

awarded to James Shylock, on the east; and we order and direct that the said Francis Tasker, and the owners and occupiers for the time being of this allotment, shall make, and for ever maintain, a sufficient, ditch and fence on the west and north sides thereof; which said allotment to the said Francis Tasker shall be held as copyhold of the manor of ——.

Declaration as to the tythe allotment to James Shylock. And we do hereby declare, that the allotment herein awarded to the Rev. James Shylock, rector, is in our judgment equal in value to one-sixth part of the said common, as directed by the said recited act to be first set out; and we declare, that the said allotment shall be a full compensation and satisfaction for all and all manner of tithes, both great and small, and all ecclesiastical dues and payments whatsoever (except such as in the first recited act is mentioned to be reserved) of him the said James Shylock arising, growing, increasing or payable for and in respect of the said common in the township of B——.

Declaration as to the allotment to John Blakelock, Esq. lord of the manor of B. And we declare, that the allotment herein awarded to John Blakelock, Esq. as lord of the manor of B——, is in our judgment equal to one-twentieth part of the said common (according to the directions of the act for inclosing the same); and we declare the said allotment to be a compensation to the said John Blakelock, lord of the manor, according to the directions of the said first recited act, for his several and respective estates and interests in and to the soil of the said common or waste lands and grounds, as lord of the soil or freehold thereof respectively.

Declaration as to allotments of the residue of the common. And we do declare, that the allotments herein awarded to the several owners and proprietors of ancient frontsteads, having right of common upon the said common or waste grounds (by the said first herein recited act) intended to be inclosed within the township of B——, are equal in value to one half the remainder thereof (viz. after the tithes, manorial right, &c. are set out), and that the same are allotted according to the number of their respective frontsteads; and that the residue of the said common or waste grounds is allotted unto and amongst all and every the owners and proprietors of ancient inclosed lands within the said township of B——, in due proportions, according to the true annual value of each proprietor's ancient inclosed lands therein.

And lastly, we do, in obedience to the said act, order and appoint that the ditches to be made in the lands and grounds by the said act directed to be divided and inclosed, shall be one

foot distant from the fence or quick-wood, and of the width of three feet six inches, and of sufficient depth to carry off the water freely; and shall be for ever hereafter repaired, cleansed, scoured and maintained by the respective owners and occupiers for the time being of the lands whereunto the said ditches respectively appertain or belong, and by whom the same ditches are to be made; and we declare, that such ditches shall from henceforth be deemed and be public water courses. In witness whereof, we, the said commissioners, have hereunto set our hands and seals, this day of in the year of the reign of King George the Fourth, and in the year of our Lord .

T. H.
R. B.

And thus I might proceed to the division of various commons, open fields, &c. whose deductions and calculations would run to a very great length, such as the division of a common and open fields belonging a township whose lands are of two or three tenures, and subject to two or three sorts of tithes; but since such proceedings cannot be laid down and explained within narrow limits, I shall not trouble the reader with them here, especially since the method of managing them is exactly the same as is described above, due regard being had to the act, &c.: therefore, if the reader only understands what has been already laid down on the subject, he will be able of himself to surmount any other difficulty (without any assistance whatever) that may afterwards happen to fall in his way.

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